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Methodology for Constructing Risk Free Zero-Coupon Yield Curve in the Eurozone: a Proposal on Standard

EFFAS-EBC

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Content

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Executive summary

Construction of a zero-coupon yield curve (or **spot** yield curve) for the Eurozone countries is one of the key problems of financial engineering and risk management[1], which, however, does not have a universal commonly accepted solution so far. A similar task for the risk-free dollar yield curve is definitely easier, since all US federal government notes and bonds have the same credit rating (although the liquidity may vary). The main difficulty of our case is that the notes we need to analyze are of different credit quality.

Currently, there are 12 Eurozone countries who issued Euro-nominated debt. These are Austria, Belgium, Finland, France, Germany, Greece, Italy, Ireland, Luxembourg, the Netherlands, Portugal and Spain. The number of outstanding issues varies from one for Luxembourg and three for Ireland to around fifty (Germany, Italy). The major issuers are Italy, France and Germany. The credit quality of the obligors varies substantially, too. German notes have the highest credit rating, whereas the rating of the Greek debt is the lowest in the Eurozone.

The aim of this paper is to suggest a methodology that could become the EFFAS-EBC standard for constructing the zero-coupon spot yield curve based on the bond market data (prices, quotes, bid-ask spreads etc.) available for government notes (medium and long-term) .

The general prerequisite for the methodology is that it should treat issues of all obligors equally, on a common basis. Thus, a methodology that would derive the risk-free yield curve from the German bond quotes[2] – due to the fact that German debt has the highest credit rating in the Eurozone – would be unacceptable as the standard. Other prerequisites are:

- universality,
- transparency and simplicity,
- compliance with the best practice,
- robustness (i.e. low sensitivity to the choice between parametric and spline fitting),
- adaptability (i.e. accommodation to the changing market conditions).

We also discuss the potential applicability of some additional data, such as ISDAFIX benchmark for fixed rates on interest rate swaps on EURIBOR and EURIBOR rates.

We test the methodology empirically, directly using Eurozone government debt market data. Credibility of the main idea of the methodology is also tested indirectly using Swiss market municipal and government data.

An overview of the Eurozone government debt market is presented in the Appendix.

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[1] *Van Deventer D.R.* Van Deventer Insights - Evaluating Yield Curve Smoothing Techniques with Implications for Credit Spreads – www.riskcenter.com, 2004.
[2] That is what the financial engineers often do in practice.

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Description of the methodology

Distinction of zero-coupon yield curve from yield curve

Financial theory and practice suggest two alternative concepts that reflect the term structure of interest rates: these are the yield curve and the zero-coupon yield curve, or spot rate curve.

The use of the former concept is generally associated with computational simplicity and relative proximity of the yield to maturity and zero-coupon curves. Usually one of the variables describing a particular bond is its maturity and other is its yield to maturity (YTM), so that a set of bonds can be mapped into a set of points on the plane with maturity – YTM coordinates. To obtain a yield curve, which constitutes a smooth curve passing near the aforementioned set of points, a parametric or non–parametric method of fitting is used. This gives a general picture, useful for understanding the evolution of the bond market. This concept was used for establishing EBC Standard for Constructing the Eurozone Benchmark Yield Curve, proposed by Klepsch and Golden (2003)[3]. Note that more adequate modification of the yield curve definition uses the duration instead of the bond maturity.

The second concept of the term structure of interest rates provides a better grasp of the economic sense, since yield to maturity is not relevant for bond pricing[4] and, more generally, for PV calculation. However, it is important that yield curve gives market practitioners a common reference point to price the present value of money accurately, especially for financial engineering and risk management applications. That is why a standard for risk free zero-coupon yield curve must be established as an important complement to the Eurozone Benchmark Yield Curve.

However, practical implementation of this concept requires a more elaborate technique of yield curve estimation based on the coupon bond market data. Parametric and non-parametric (spline) methods are used for this purpose.

The difference between the zero-coupon yield curve and the yield curve is known as the coupon effect. Some empiric studies of the government bond samples suggest that the coupon effect is insignificant and can be disregarded when estimating the term structure of interest rates[5]. Other researches illustrate the illegitimacy of using the yield to maturity curve for estimation of the term structure of interest rates by pointing out individual substantial deviations of the zero-coupon and yield curves[6].

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[3] *KlepschT., Golden C.* Constructing the Eurozone Benchmark Yield Curve, Working paper, March 2003
[4] see, for example, *Fabozzi F*. Bond Markets, Analysis and Strategies // Prenticehall,2000
[5] see, for example *Malkiel B*. “The term structure of interest rates” // Princeton: Princeton University Press, 1966; or Nelson C. “The term structure of interest rates” // New York: Basic Books, 1972.
[6] Echols M., Elliott J. “A Quantitative yield curve model for estimating the term structure of interest rates” // Journal of Financial and Quantitative Analysis, # 11, 1976, pp. 87-104; Conard J., Frankena M. “The yield spread between new and seasoned corporate bond yields” // National Bureau of Economic Research, 1969

A well-grounded justification of significance of the coupon effect is presented in Livingston and Jain (1987)[7]. Under particular circumstances, the coupon effect results in a change in the shape of the forward curve, thus the estimate of the term structure of interest rates based on the yield curve turns out to be biased. In general, the higher the volume of cash flows in the earlier periods of a bond's circulation, for example, in case of gradual amortization, say for serial (installment) bonds, the larger the deviation between the yield to maturity and the zero-coupon yield (provided that there are no equal yields to maturity). Basically, if a developed market of discount bonds were available, the yield curve could be constructed using solely the data from this market, without referring to the market of coupon bonds. However, this would require that discount bonds with maturity of over one year are present on the market, which is hardly observable in practice, except for the instance when a liquid strip market is available.

Modified yield curve, i.e. a curve using YTM and duration, can be used as the first approximation of the zero-yield curve.

Relation between discount function and spot yield

It is strongly suggested that the standard establishes continuous compounding as the conventional relation between the discount function and the spot yield:

$$d(t) = exp(-t\, r(t)),$$

where $d(t)$ is the discount factor and $r(t)$ is spot rate at maturity t.

This is the usual convention for derivatives pricing. Continuous compounding is more consistent when defining the relation between discount factors and spot rates, because in this case the bond duration, up to the sign, represents the price sensitivity to the parallel shifts of spot yield curve and this is **invariant with respect to the shape of the spot yield curve**.

Best practice procedure for credit spread evaluation

Van Deventer (2004) suggests the following best practice procedure for calculation of the credit spreads on a 'credit model independent basis'. Assume that there are M payments the ABC Company bond and that all observable non-call U.S. Treasuries are used to create a smooth, continuous Treasury yield curve using the techniques we describe below. Then the continuous credit spread on the ABC bond can be constructed like this:

1. For each of the M payment dates on the ABC Company bond, calculate the continuously compounded zero coupon bond price and zero coupon yield from the U.S. Treasury smoothed yield curve. These yields will be to actual payment dates, not scheduled payment dates, because the day count convention associated with the bond will move scheduled payments forward or backward (depending on the convention) if they fall on weekends or holidays

[7] Livingston M., Jain S. "Flattening of bond yield curves for long maturities" // Journal of Finance, vol.37, # 1, 1982, pp. 157-167.

2. Guess a continuously compounded credit spread of *x* that is assumed to be the same for each payment date.

3. Calculate the present value of the ABC bond using the M continuously compounded zero coupon bond yields $y(t) + x$, where $y(t)$ is the zero coupon bond yield to that payment date on the risk free curve. Note that $y(t)$ will be different for each payment date but that *x* is assumed to be constant for all payment dates.

4. Compare the present value calculated in Step 3 with the value of the ABC bond (price plus accrued interest) observed in the market.

5. If the theoretical value and observed value are within a tolerance, then stop and report *x* as the credit spread. If the difference is outside the tolerance, improve the guess of *x* using standard methods and go back to Step 3.

It should be observed that:

- Spreads calculated in this manner should be confined to non-callable bonds or used with great care in the case of callable bonds.
- The suggested approach also disregards the liquidity premium, which can be rather substantial on an illiquid market. To be precise, the resultant spreads consist of both credit risk premium and liquidity premium. In particular, the reduced form modeling approach of Duffie and Singleton[8] and Jarrow[9] has the power to extract default probabilities and the 'liquidity premium' (the excess of 'credit spread' above and beyond expected loss) from bond prices and credit default swap prices.
- A substantial simplifying assumption of the suggested approach is the absence of term structure of credit spreads. However, a corresponding modification can be introduced into the method, as suggested below.

Obviously, this procedure relies on a known risk-free zero yield curve. The evident benchmark for the dollar denominated debt market is the U.S. Treasuries market. Thus, there are no problems with construction of the relevant yield curve.

The Eurozone is a different issue, since there is currently no standard for determination of the risk-free zero yield curve. Our main idea is to use the best practice procedure of calculation of credit spreads described above in order to construct the Eurozone risk-free zero yield curve by solving the inverse problem. That means that we should choose a risk-free zero yield curve so that the credit spreads relative to this curve were estimated with most precision. It is evident that this problem will have non-unique solution, and the curve can be accurately determined up to an additive constant. Thus, we will denote the corresponding family of parallel shifts as the **relative** risk-free spot yield curve.

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[8] *D. Duffie and K. Singleton* Modelling Term Structures of Defaultable Bonds / Review of Financial Studies, 12(4), 197-226, 1999.

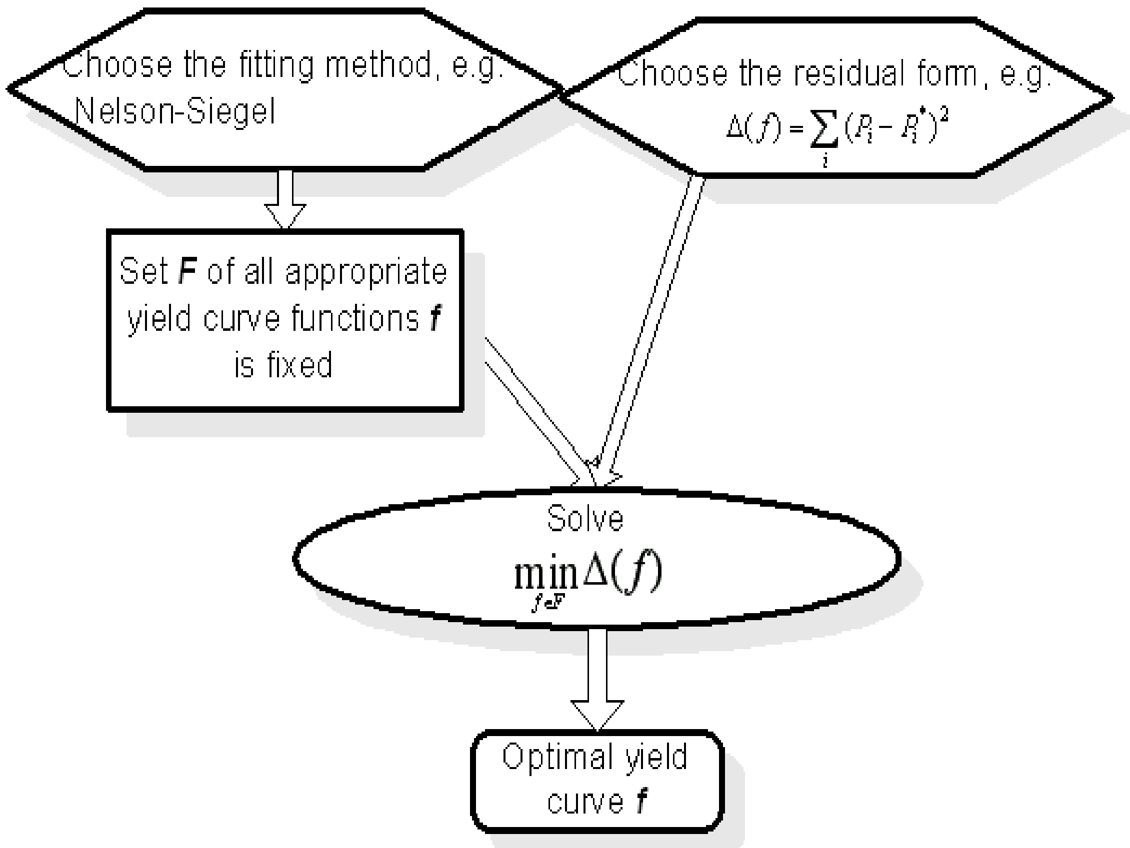
[9] *R. Jarrow* Default Parameter Estimation using Market Prices / Financial Analysts Journal, 2001.

Standard algorithm for yield curve fitting

For simplification, let us consider that the parametric approach, such as the one suggested by Nelson and Siegel (1987)[10] or Svensson (1993)[11], is employed for yield curve fitting. Of course, implementation of advanced nonparametric techniques based on smoothing splines[12] is also possible. However, given the robustness prerequisite, the selection of the fitting technique should not have any substantial impact on the fitting results.

The standard algorithm of yield curve fitting in case of homogenous credit quality of the set of bonds is provided on Figure 1Figure 1. The figure is provided to illustrate the alterations that are required for the solution of the main problem.

Figure 1: Standard algorithm of yield curve fitting



[10] *Nelson C, Siegel A.* Parsimonious Modeling of Yield Curves / Journal of Business 00,473-489, 1987.
[11] *Svensson L.E.O.* Estimating forward interest rates with extended Nelson-Siegel method / Sveriges Riksbank Quarterly Review #3, 1993.
[12] For instance, see *Adams K., Van Deventer, D.* Fitting Yield Curves and Smooth Forward Rate Curves with Maximum Smoothness / Journal of Fixed Income, 1994 or *Smirnov S., Zakharov A.* A Liquidity-Based Robust Fitting of Spot Yield Curve: Splines Guaranteeing Positive Forward Rates - www.effas-ebc.com, 2003.

Methodology of relative risk-free spot yield curve and credit spread construction – a proposal on the standard

In case that the set of bonds consists of groups with different credit quality (in our case a group is comprised of bonds of a particular state), the basic algorithm suggests that a specific yield curve for each group is derived through a parallel shift of the base curve from the given family of the relative risk-free spot yield curves. The fitting is carried out in respect to both the base curve and the shift parameters. Here, a shift would correspond with the credit quality of a particular group[13]. One of the grounds is strong empirical evidence for a global factor that mainly represents the average level of the yield spreads, and for a country, the specific factor for each issuer[14].

It should be once again emphasized that in this specification the minimization problem has a family of solutions. Given a certain set of optimal curves, a simultaneous parallel shift will result in another optimal set. Thus, relative spreads (for instance, spreads relative to a certain country) are uniquely defined.

We suggest that the resultant base curve defined up to a parallel shift is regarded as the **relative risk-free spot yield curve for the Eurozone.**

The proposed methodology can be improved in order to take into the account the term structure of credit spreads. To capture this second order effect, an additional parameter of the curves should be introduced, such as a (permanent) slope parameter individual for each country. In this case, the relative risk-free spot yield curve for the Eurozone will be accurate within two parameters: the level and the slope of the curve. This is consistent with empirical finding of Sorge and Gadanecz[15]: the term structure of credit spreads can reasonably be approximated by a linear positive function of maturity. On average, the term structure of credit spreads for investment grade bonds appears upward-sloping, in line with the intuition that lenders should get a higher compensation for being exposed to risk for a longer period of time. This is consistent with the analyses in the papers of Jones et al[16], Sarig and Warga[17] and He et al[18].

However, it seems unnecessary to double the number of estimated parameters of term structure of credit spreads. For example, we can use slope parameter additionally to level

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[13] As it was previously noted, we disregard the liquidity premium, thus assuming that the Eurozone sovereign debt markets are sufficiently liquid. Otherwise, the method should be implemented based on the highly liquid benchmark issues only.

[14] *Geyer A., Kossmeier S., Pichler S*. Empirical Analysis of European Government Yield Spreads, working paper,Vienna,March 2001

[15] *Sorge M., Gadanecz B*. The term structure of credit spreads in project finance, Bank for International Settlements, Monetary and Economic Department, August 2004

[16] *Jones, E Mason, S and E Rosenfeld* (1984): "Contingent claims analysis of corporate capital structures: An empirical investigation," Journal of Finance 39, 611-627.

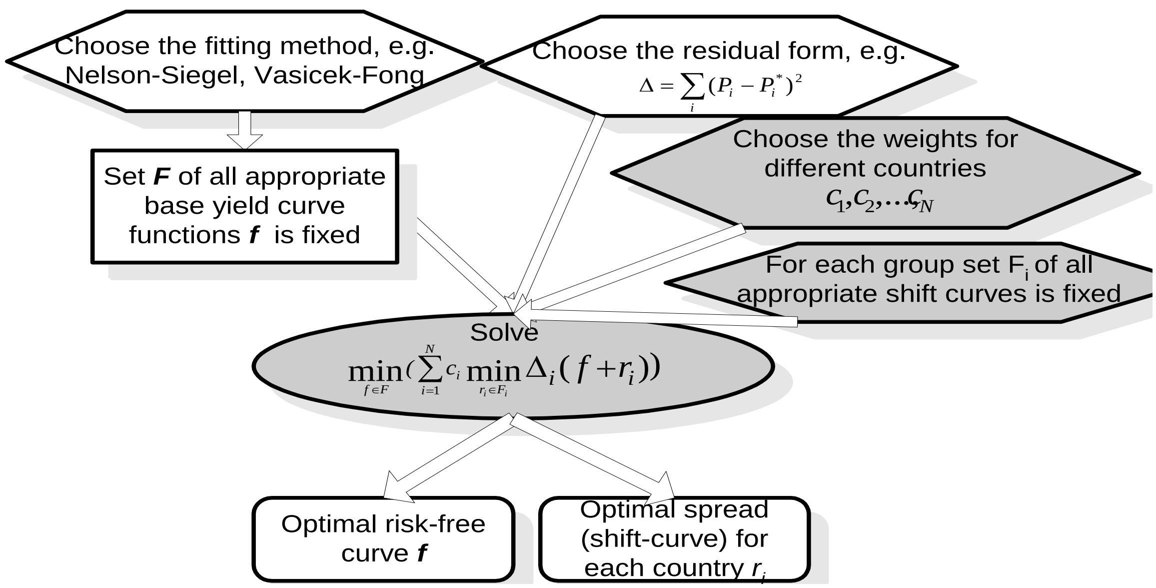
[17] *Sarig, O and A Warga* (1989): "Some empirical estimates of the risk structure of interest rates", Journal of Finance, 44(5), pp 1351-60

[18] *He, J W Hu and L Lang* (2000): "Credit spread curves and credit ratings", Chinese University of Hong Kong working paper.

parameter only for those countries where the clear manifestation of sloping credit spreads is observed.

In principle, other variation parameters, such as curvature parameter, can be added into the model. A schematic representation of this algorithm is provided on Figure 2.

Figure 2: Enhanced algorithm of yield curve fitting



In order to estimate the absolute, rather than the relative, risk-free spot yield curve for the Eurozone, it is suggested that an additional procedure (and possibly, additional data) is used to determine the level (and, in case of an advanced specification of the model, the slope) of the risk-free yield curves.

The appropriateness of using Euribor (Euro Interbank Offered Rate) swap rates as such additional data is discussed below.

The immediate solution of the minimization problem of the baseline yield curve and the spreads between the estimated and actual market prices can be used as the algorithm of practical implementation of the proposed methodology. However, the results of this approach are not independent from the implemented fitting technique, thus the existent standard fitting modules cannot be directly built into the algorithm. A simple risk-free spot yield curve construction algorithm based on implementation of standard fitting modules is suggested below.

Algorithm of risk-free spot yield curve construction: an alternate procedure,fitting – credit spread adjustment

The proposed algorithm of solving the following problem:

$$\min_{f,r_i}\sum_{i=1}^{N}c_i\left(\sum_{j=1}^{K_i}\left(P_{i,j}(f+r_i)-P^*_{i,j}\right)^2\right)$$

is based on separate fitting and spread optimization.

Preliminary stage

Let us assume that the spreads for all issuers equal zero.

Main Stage

1) The yield curve fitting is performed for the whole set of bonds in consideration of the current spreads, i.e. a solution is found for the problem of constructing a spot yield curve that would minimize the deviation between the estimated theoretical and actual market prices of the bonds if shifted by a credit spread specific for each country.

Formally, the problem can be defined as:

$$\min_{f}\sum_{i=1}^{N}c_i\left(\sum_{j=1}^{K_i}\left(P_{i,j}(f+r_i)-P^*_{i,j}\right)^2\right),$$

where N denotes the number of governments;

K_i denotes the number of bonds issued by the *i*-th government in the sample;

$P^*_{i,j}$ denotes the market price of the *j*-th bond of the *i*-th country;

$P_{i,j}(f+r_i)$ denotes the theoretical price of the *j*-th bond of the *i*-th government, estimated as the discounted value of the future cash flows of the bond derived from the yield curve f, shifted by a spread r_i specific for the *i*-th government.

Formally:

$$P_{i,j}(f+r_i)=\sum_k C_k^{i,j}\exp(-(f+r_i)t_k),$$

where $C_k^{i,j}$ - denotes the amount of the *k*-th payment on the *j*-th bond of the *i*-th country;

t_k - denotes the time to *k*-th payment.

The selection of a particular fitting technique in this instance is irrelevant, both parametric and spline fitting can be implemented. Here, the alterations introduced into the fitting algorithm are minimal and correspond to the notion that in an adjusted fitting algorithm, each bond can have a specific spread.

This alteration can be fulfilled by adjusting the future payments: the *k*-th future payment S_k^i on the i-*th* bond is replaced by a discounted payment $S_k^i\exp(-r_i t_k)$, where r_i denotes the spread specific for the *i*-th bond; t_k denotes the time to *k*-th payment.

Once such adjustment of the payments is performed, an arbitrary standard fitting technique can be implemented without any further adjustments.

2) The optimal spread is estimated for each country, i.e. the shift of the baseline curve constructed on the previous step is found for each country, so that it minimizes the deviation between the estimated theoretical and actual market prices of the bonds of this country.

Formally, a single-dimension optimization problem is solved for each country:

$$\min_r \sum_{i=1}^{K} \left(P_i(f+r) - P_i^* \right)^2 ,$$

where K denotes the amount of a country's bonds included in the sample;

f denotes the baseline curve constructed at the previous step;

P_i^* denotes the market price of the *i*-ой облигации

$P_i(f+r)$ denotes the theoretical price of the *i*-th bond estimated by discounting the future payments on a bond by the yield curve f shifted by a country-specific spread r.

3) Return to step 1.

In practice, convergence is achieved in up to five iterations of steps 1) – 3), thus the numerical efficiency of the suggested algorithm would differ from the efficiency of the standard fitting algorithm roughly as many times. Yet it should be observed that implementation of the algorithm does not ensure convergence. It can be reduced to the convergence of Seidel Iterative Method with respect to equation gradient of minimized expression:

$$\sum_{i=1}^{N} c_i \left(\sum_{j=1}^{K_i} \left(P_{i,j}(f+r_i) - P_{i,j}^* \right)^2 \right)$$

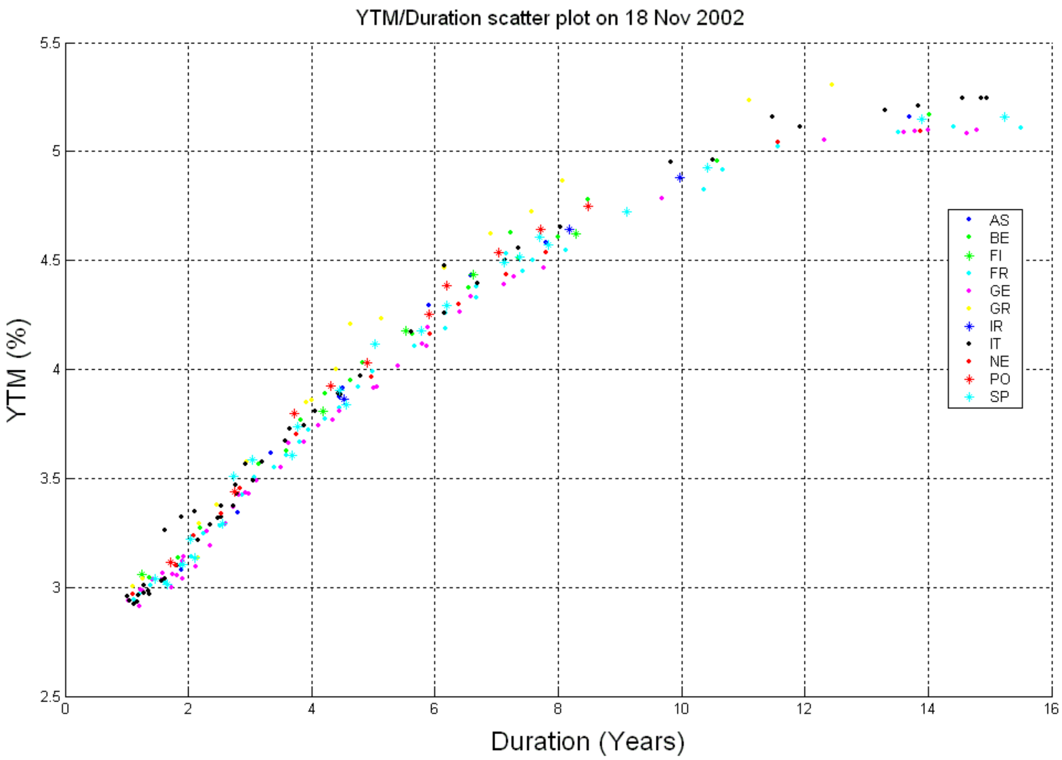
The sufficient conditions of convergence of Seidel Iterative Method are known and can basically be verified for this particular fitting technique. Moreover, even in case of convergence we cannot be sure that it converges to the global minimum, because in pinciple there can be several local minimums and the method is converging to one of them. However, the computations performed in course of the study indicate that the results of the procedure of alternate fitting – credit spread adjustment and the results of the global minimization procedure are similar.

Empirical data analysis and interpretation of results

Relative risk-free spot yield curve and credit spread construction for Eurozone

Currently, the suggested methodology (assuming flat shift-curves) was applied to the Eurozone sovereign debt market as of November 18, 2002 (since this is the only data for which the data was available). The market situation for that date is presented on Figure 3.

Figure 3: YTM and duration of the Eurozone sovereign debt issues, 18.11.2002



The fitting was performed using the parametric approach suggested by Nelson and Siegel (1987) and by sinusoidal-exponential spline method[19]. The results of the two approaches were nearly identical, which indicates that the suggested methodology is not sensitive to the selection of the fitting approach. The risk-free yield curves derived using the two different approaches are presented on Figure 4 below.

It is interesting to compare obtained spot yield curves to the benchmark yield curve computed by Klepsch and Golden (2003)[20] – the similarity is clear.

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[19] *Smirnov S., Zakharov A.* A Liquidity-Based Robust Fitting of Spot Yield Curve: Splines Guaranteeing Positive Forward Rates - www.effas-ebc.com, 2003.
[20] *Klepsch T., Golden C.* Constructing the Eurozone Benchmark Yield Curve, Working paper, March 2003.

Figure 4: Risk-free spot yield curves for Eurozone sovereign debt, 18.11.2002

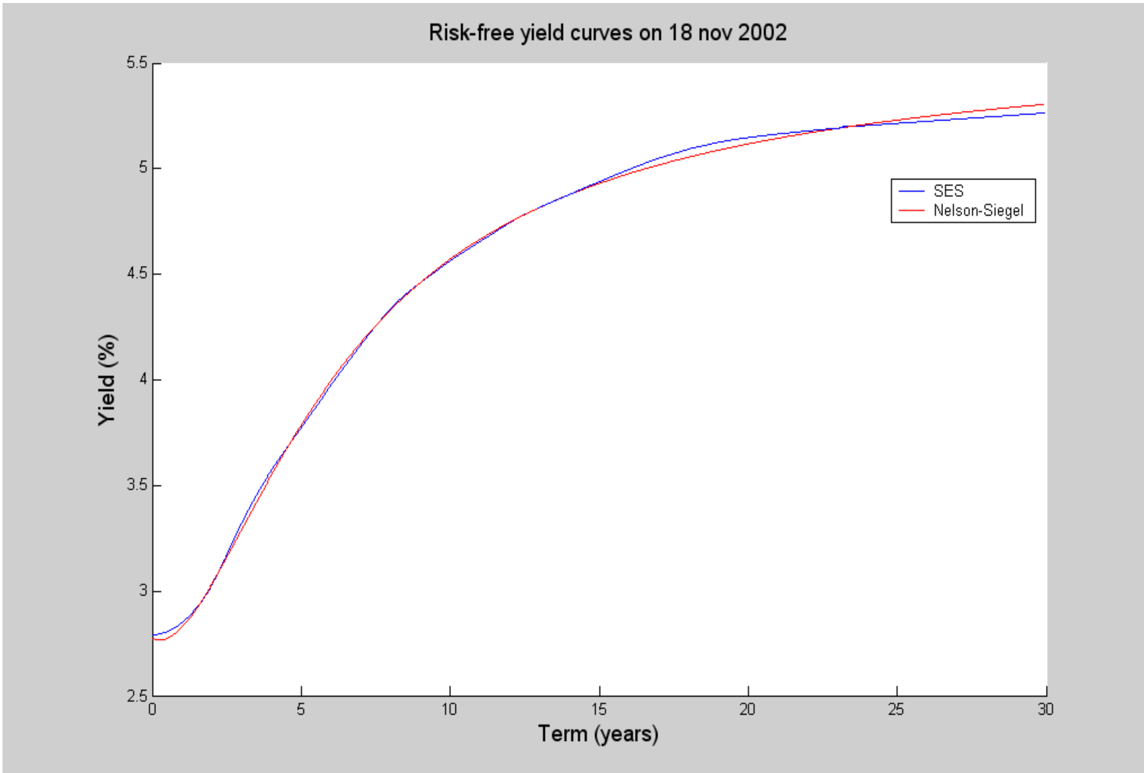

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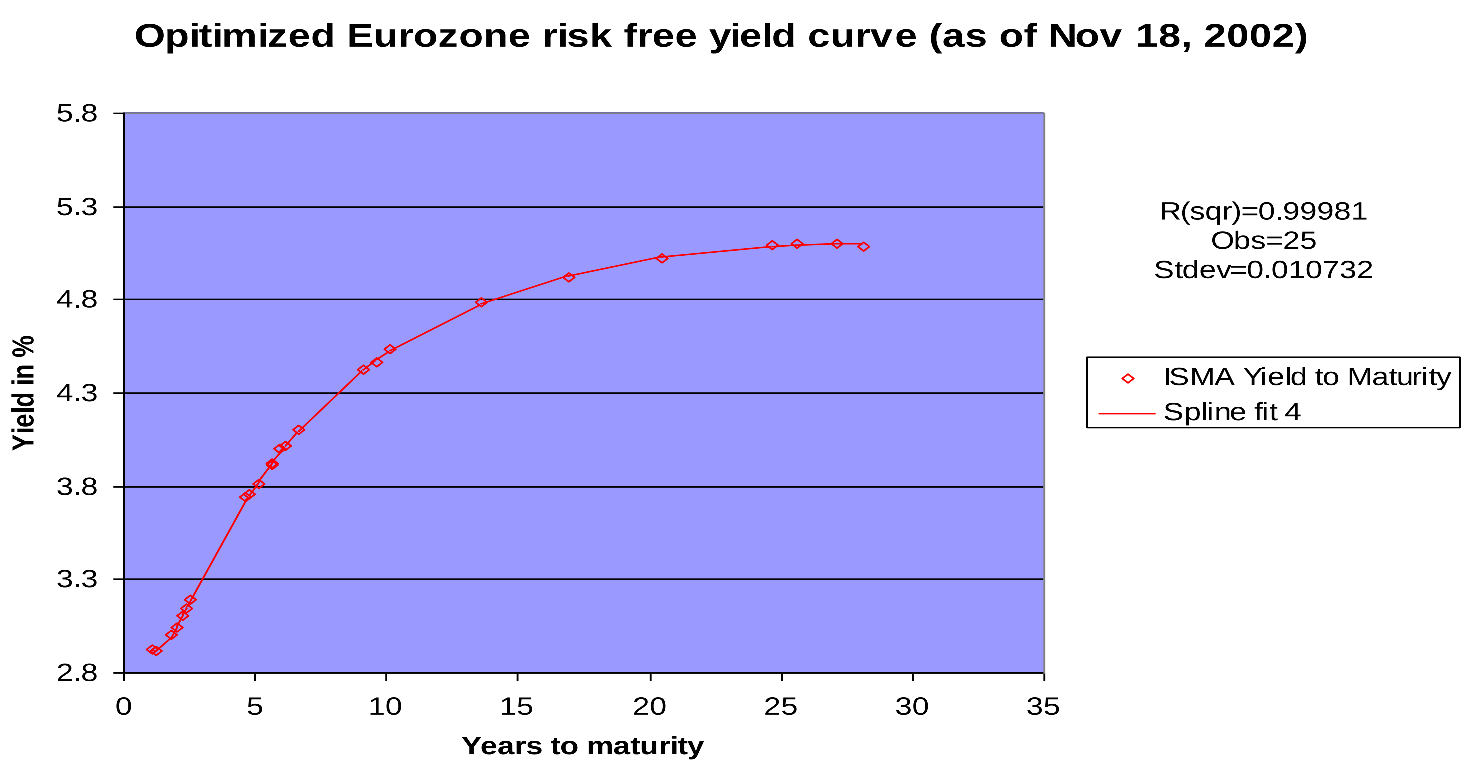
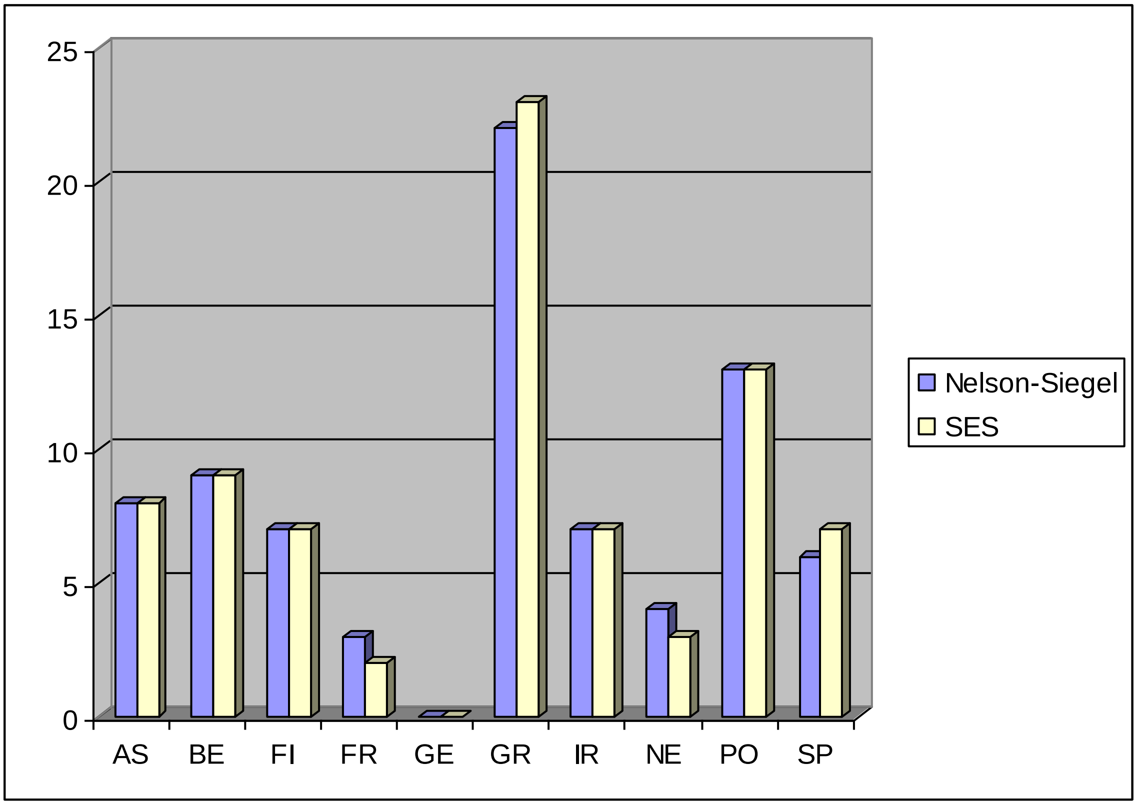
Figure 5: Benchmark yield curve for Eurozone sovereign debt, 18.11.2002



The correspondent relative country spreads are presented on Figure 6 (the spread for Germany was assumed to be zero in order to remove the ambiguity of the solution arising from shifts).

Figure 6: Relative country spreads 18.11.2002



The figures above demonstrate the average and maximum yield to maturity estimation errors for the two curves, where the first curve is estimated solely on the national data and the second curve is derived by a parallel shift of the Eurozone risk-free yield curve.

We also present several figures comparing the national risk-free yield curves estimated solely on the national data and derived by a parallel shift of the Eurozone risk-free yield curve. Overall, the two curves look very likely for all countries but Greece. The results for Greece can be improved by introducing a simple linear slope component into its credit spread term structure.

Figure 7: Average YTM estimation errors of Nelson-Siegel fitting (in bp), 18.11.2002

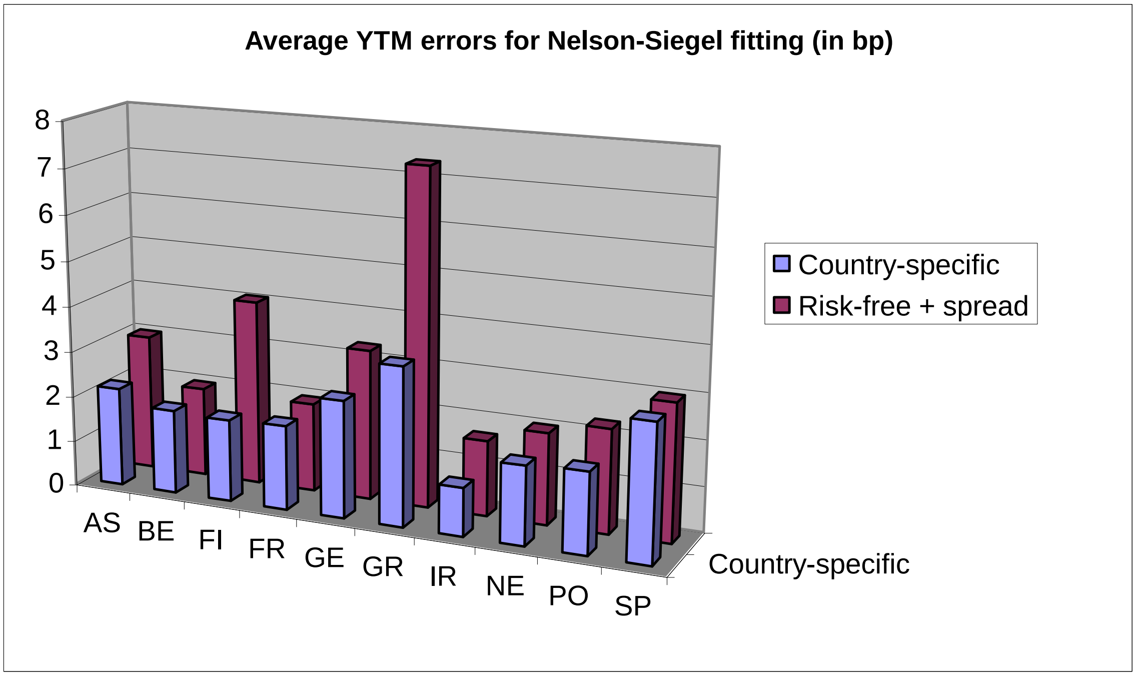


Figure 8: Maximum YTM estimation errors of Nelson-Siegel fitting (in bp), 18.11.2002

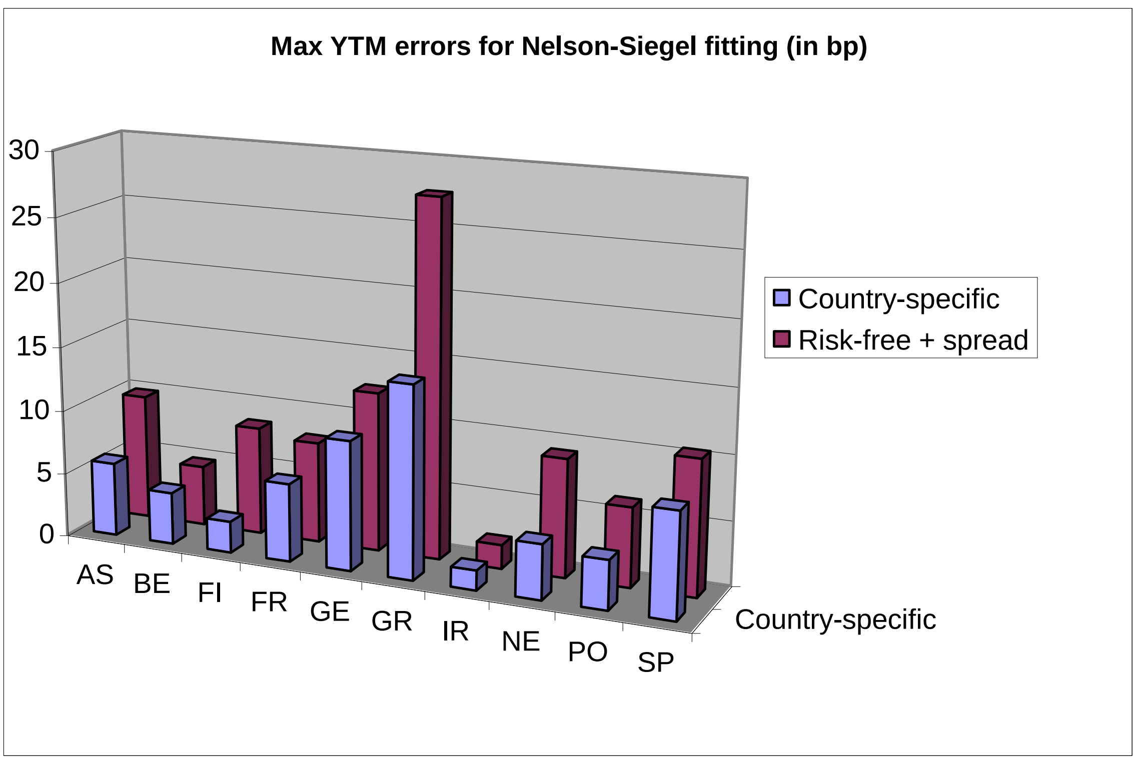


Figure 9: Spot yield curves for France, 18.11.2002

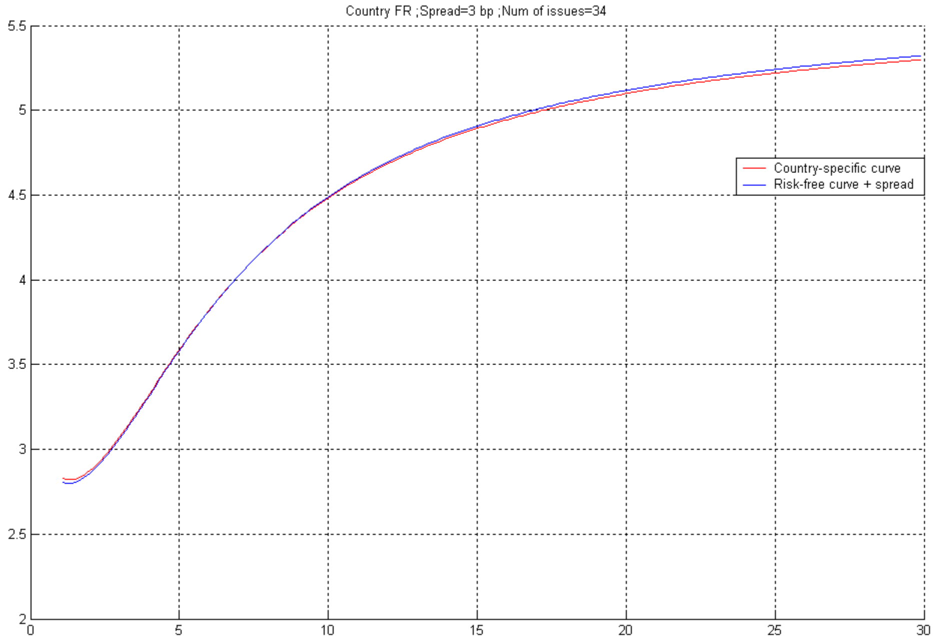


Figure 10: Spot yield curves for Germany, 18.11.2002

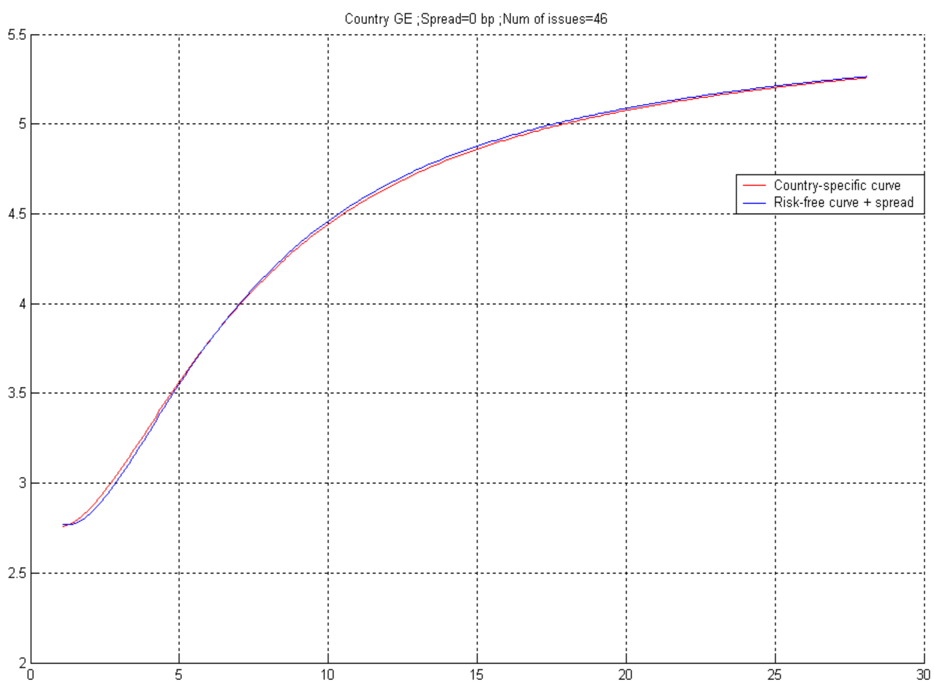


Figure 11: Spot yield curves for Netherlands, 18.11.2002

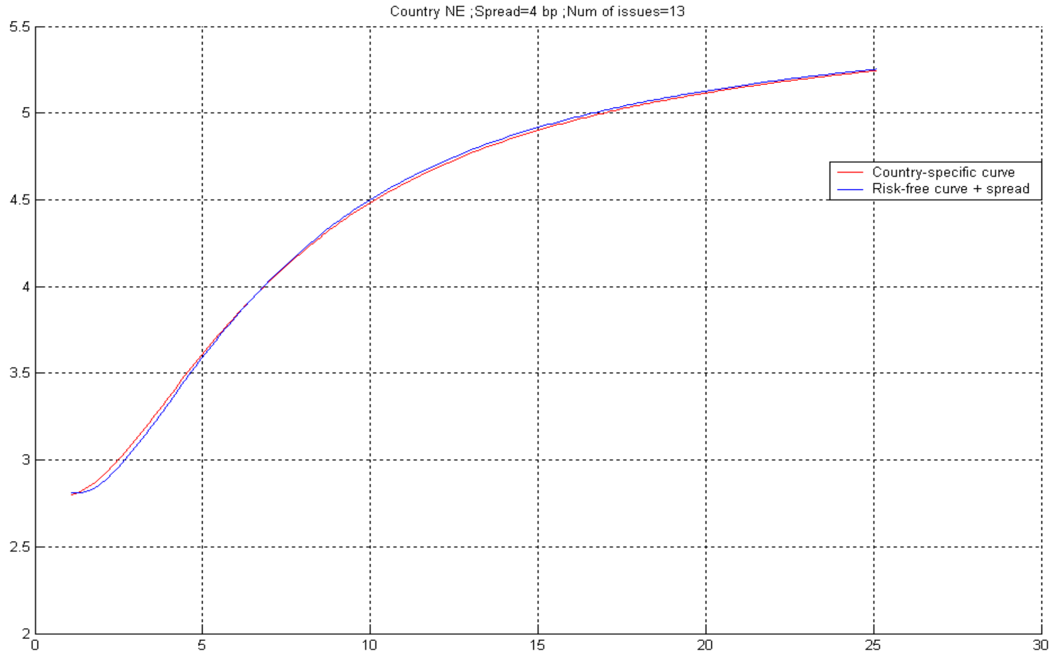
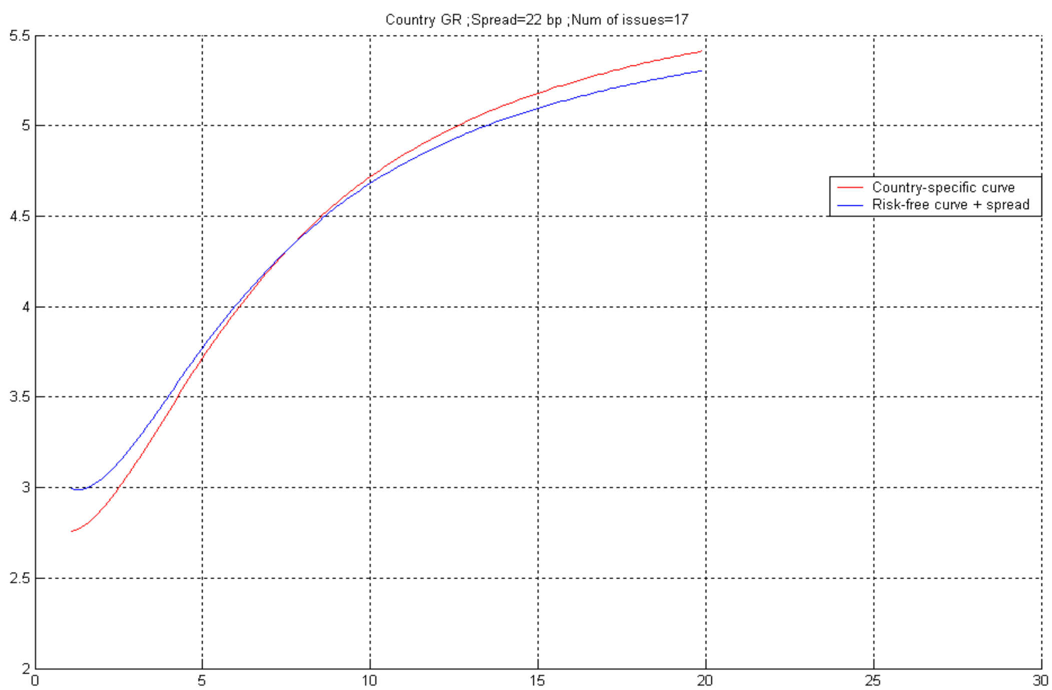


Figure 12: Spot yield curves for Greece, 18.11.2002



Estimation of the level (and, potentially, the slope) of the risk-free spot yield curve

The parameters of relative risk-free spot yield curve are subject to estimation using initial bond data or additional data like Euribor swap rate. We believe that one of the possible solutions could be adaptation of the procedure, proposed by Klepsch and Golden (2003)[21], namely the idea of eliminating the bonds with yields lying above the fitted curve. The economic interpretation of this procedure is to find the lowest available yield over the entire curve of the Eurozone risk-free spot yield curve

The problem of using Euribor swap rate is discussed below.

Is Euribor swap rate data appropriate?

It is reasonable to choose Euribor (instead of Libor, appropriate for dollar instruments) for floating rate in the context of the stated problem. A representative panel of banks provide daily quotes of this reference rate, rounded to two decimal places that each panel bank believes one prime bank is quoting to another prime bank for interbank term deposits within the Eurozone. The selection of banks quoted for Euribor is based on market criteria. These banks are of first class credit standing. They were selected to ensure that the diversity of the Eurozone money market is adequately reflected, thereby making Euribor an efficient and representative benchmark. Euribor is quoted for spot value (T+2) and on an act/360 day-count convention.

We can now pose the question if the ISDAFIX quotes are used for calibration of the level (and potentially, the slope) of the yield curves. ISDAFIX is a leading benchmark for fixed rates on interest rate swaps worldwide, providing average mid-market swap rates for six major currencies at selected maturities on a daily basis. In particular, ISDAFIX provides rates for the Euro (EUR). ISDAFIX specification for the Euro is the following:

- Maturities – 1-10, 12, 15, 20, 25, 30 years.
- Day count (rates) – Annual 30/360 vs. 6 month Euribor.

Each ISDAFIX swap quote can be used to derive the price of a coupon bond specified above valued at par, the theoretical price of which should equal its nominal value. We can derive as many coupon bond prices as there are swap quotes (i.e. different maturities). Let us lock in a certain subset S of this set of bonds. Since we estimate the risk-free yield curve accurate within a constant (and possibly, the slope), we suggest that the position of the curve is determined by minimizing the deviation of the estimated and nominal prices of the bonds belonging to subset S.

In theory, the value of the plain vanilla interest rate swap at origination time is zero, so that the present value of the stream of cash flows for fixed leg and floating leg are the same. Because the present value of the stream of cash flows for floating leg equals the notional principal, the fixed rate payments valuation corresponds to par valuation of a risk-free bond with the corresponding payments, i.e.:

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[21] *Klepsch T., Golden C.* Constructing the Eurozone Benchmark Yield Curve, Working paper, March 2003

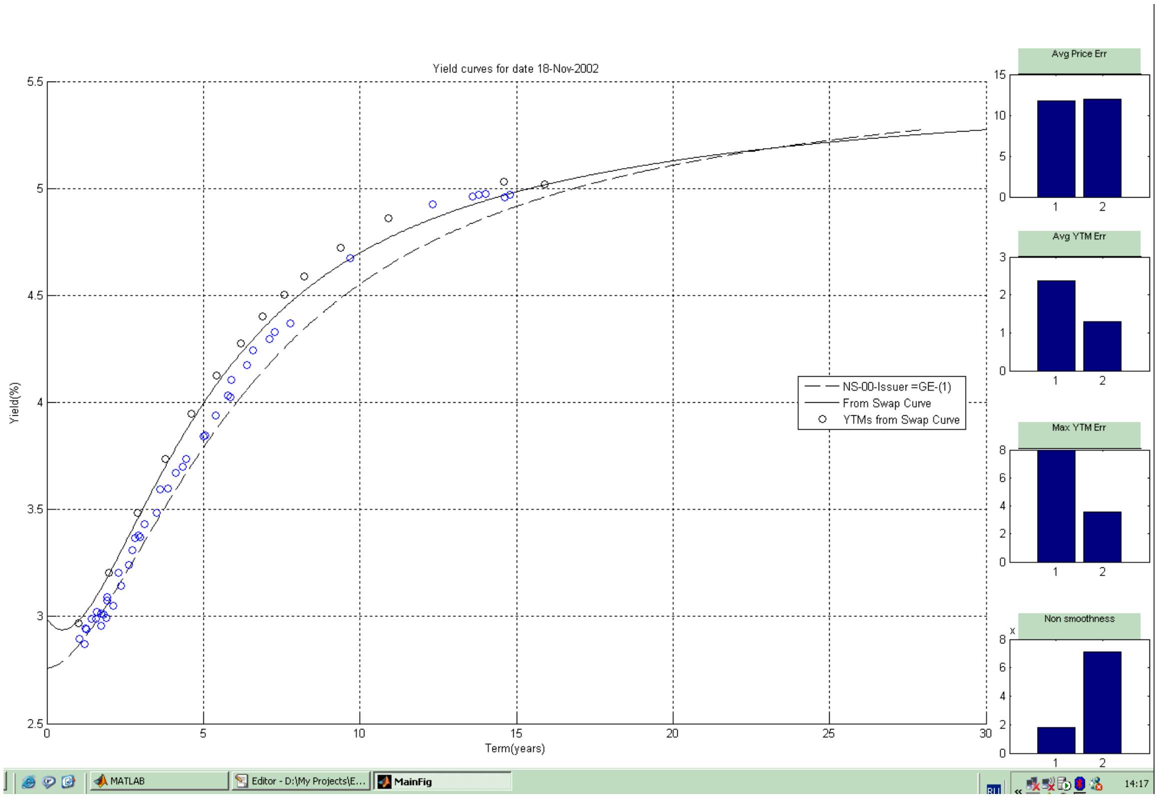
$$\Sigma c_i\, d(t_i) + d(t_n) = 1,$$

$$c_i = R\theta_i;$$

where $d(t_i)$ is the discount factor; θ_i is the proportion of annual floating rate payment, computed as a fraction of a year; and R is the fixed rate.

The calculations performed as of November 18, 2002 show, however, that the **swap curve cannot be directly used for the purposes of evaluation of yield level parameter**, since it is above the spot yield curve for Germany, see Figure 13.

Figure 13: Swap curve and spot yield curve for Germany



This result is due to the fact that the swap curve incorporates credit risk premium inherent to the interbank market, so that it leads to consistently wrong values.

Therefore, the question of evaluation of the level risk free spot yield curve for Eurozone remains open. Any suggestions concerning this issue are welcome.

Testing the methodology on the Swiss bond market data

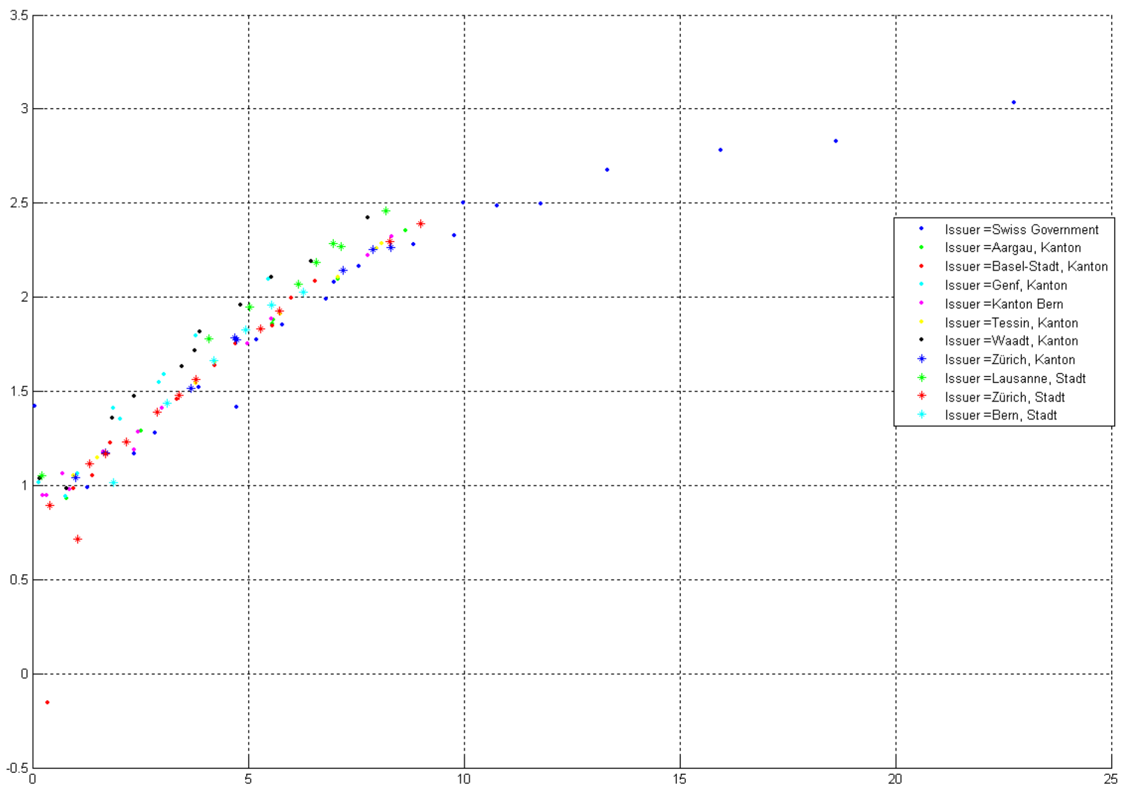
The proposed methodology was tested on the Swiss bond market data. In this instance, Swiss cantons and cities with six and more outstanding bond issues stood for the Eurozone countries.

The goals of the test were:

- to construct a relative risk free spot yield curve by applying the proposed algorithm to relative risk free spot yield curve Swiss municipal bonds; and
- to compare the resultant curve with the actual Swiss risk-free spot yield curve, i.e. the curve constructed based on the Swiss sovereign bond market data.

Data on the securities and quotes was derived from the SWX Stock Exchange publicized data. Over-the-counter market was disregarded, as no information was available on the OTC transactions. The market priced used in the computations was proxied by the closing price available for each security and each date including the dates when no transactions were made with the security (the exact algorithm of the closing price computation by the SWX is unknown). As illustrated on Figure 14, these prices can be inadequate. For instance, it is clear that one of the securities plotted on Figure 14 has a negative yield to maturity, and there are also several evident discharges.

Figure 14: Swiss bond market, 15 December2004, YTM and Duration



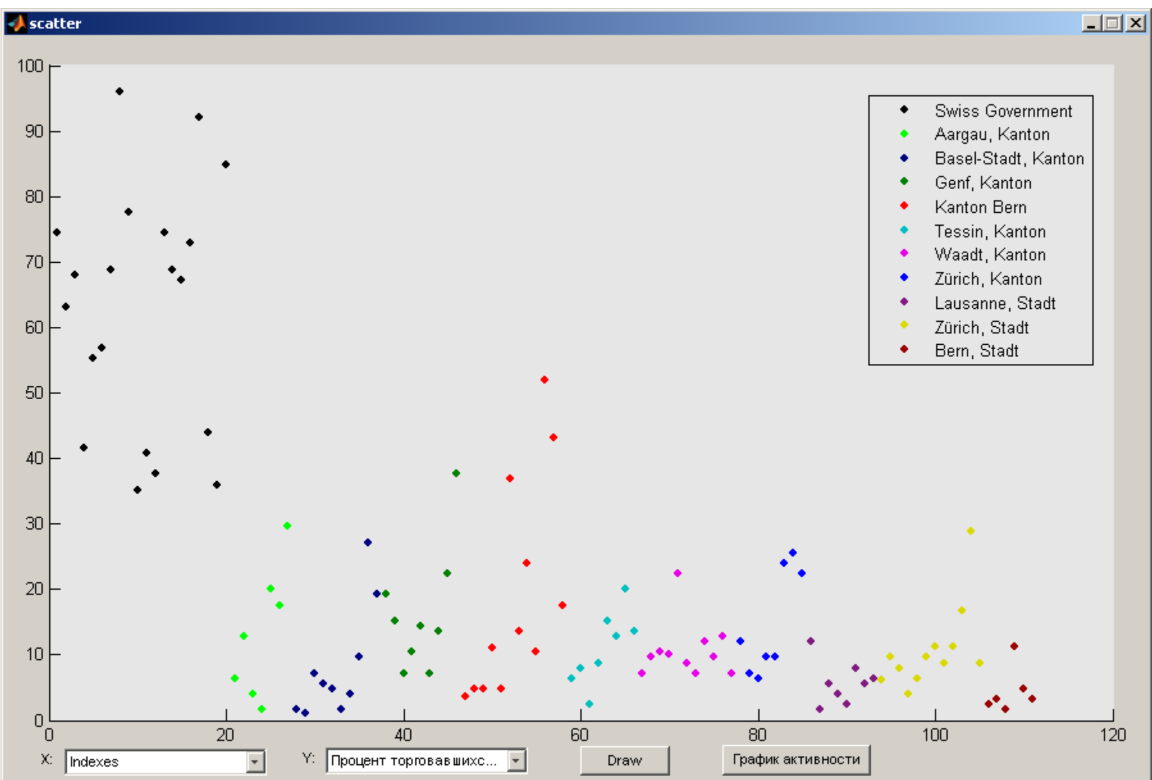
We selected a total of ten municipalities (cantons and cities) that had over six outstanding bond issues. The list of the issuers is provided on Table 1.

Table 1: Selected Swiss Municipal Bonds

Issuer	Issuer Type	Number of outstanding issues
Aargau	Kanton	7
Basel-Stadt	Kanton	10
Genf	Kanton	9
Bern	Kanton	12
Tessin	Kanton	8
Waadt	Kanton	11
Zurich	Kanton	8
Lausanne	Stadt	8
Zurich	Stadt	12
Bern	Stadt	6

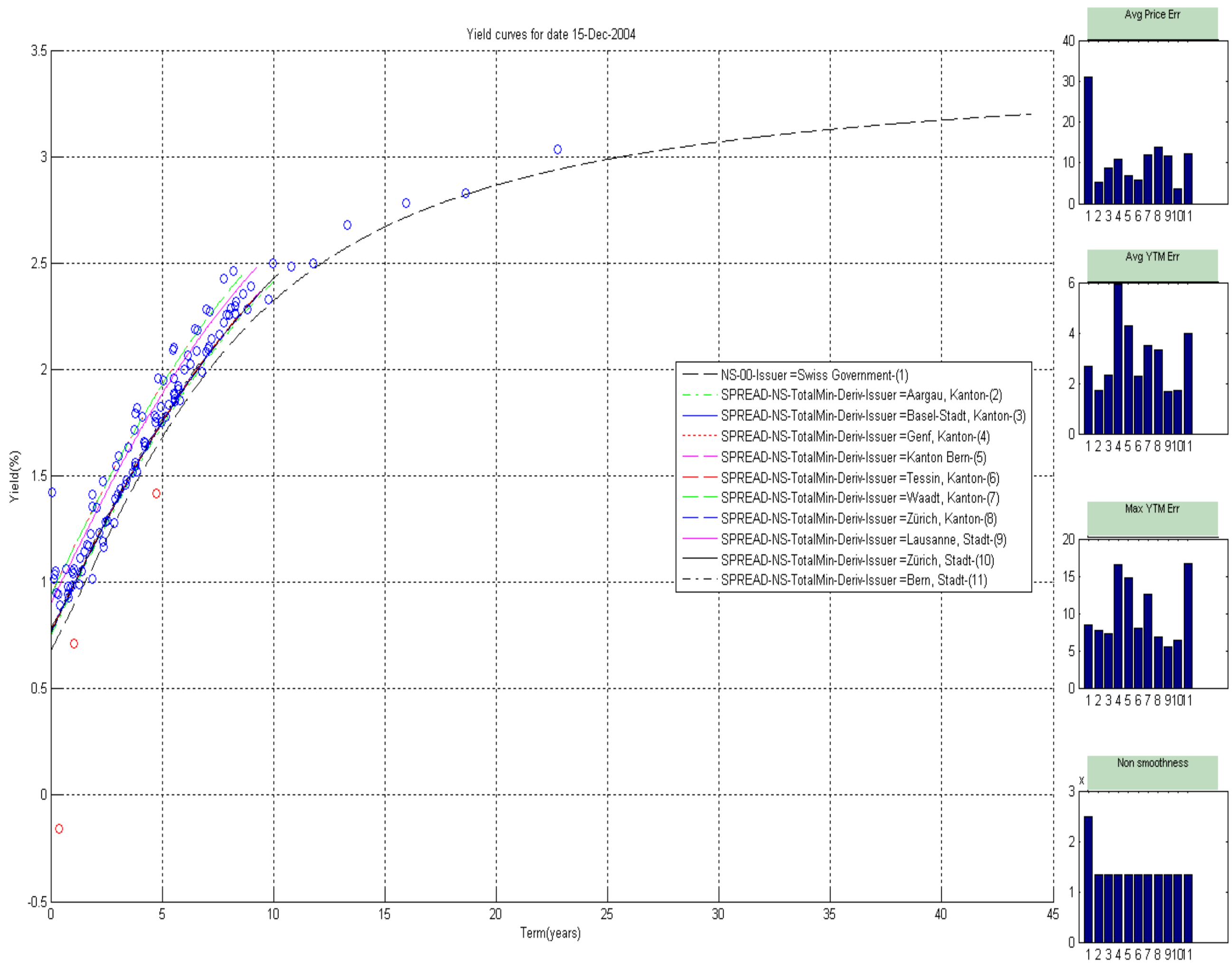
The stock exchange market of municipal bonds is highly illiquid, as illustrated by Figure 15. The x-axis of the figure denotes the bonds grouped by issuer, and the y-axis stands for the percentage of dates that the bond was traded on the SWX Stock Exchange.

Figure 15: Swiss bond market liquidity



Colored parallel curves on Figure 16 below correspond are the yield curves of individual cantons and cities resulted by a parallel shift of the baseline yield curve by a spread specific for each municipality. The black broken line is the yield curve constructed basing on the Swiss sovereign debt market data. It can be observed that these curves are nearly perfectly parallel over the ten year time horizon (ten years is the maximum time to maturity for the analyzed set of municipal bonds). The red dots denote the discharges that were automatically rejected. A fitting error of over 15 bp was used as the rejection criteria.

Figure 16: Swiss bond market relative risk free spot yield curve



Appendix: Eurozone Sovereign Debt Market Overview

It should be observed that substantial move has occurred over the last few years towards the integration of the Eurozone sovereign debt markets, which is reflected in tightening of the yield spreads. Prior to EMU, yield differentials within euro area government bonds had been determined by four main factors: expectations of exchange-rate fluctuations, different tax treatment of bonds issued by different countries, credit risk and liquidity. After the introduction of the euro, currency- related premia were eliminated by the irrevocable fixing of legacy currency pairs. With respect to taxation differences, good progress was made in harmonizing national tax treatments. Thus yield differentials are mainly generated by the credit premium and the liquidity of the market. However, differences still exist both in terms of the debt instruments, term structure and market organization. In our overview, we will try to pinpoint these differences that may have impact on the risk-free spot yield curves of the Eurozone countries[22].

Government Debt Size and Structure

Government debt market is the dominant European debt market in terms of size. The amounts outstanding have decreased relative to other segments, but it still continues to represent nearly half the Euro area bond market. A share of 70% of this market is provided by only three countries: Italy, France and Germany. A further share of 20% comprises the government issues of Spain, Belgium and Netherlands, while Austria, Finland, Portugal, Greece, Luxemburg and Ireland account for 10% of the sovereign issues outstanding.

The Table 2 presents the volumes of the outstanding debt of the Eurozone governments and the correspondent long-term debt ratings. The relative level of the total debt varies from as low as 20% (Ireland) and 33.5% (Finland) to as much as 100% (Greece). The average debt to GDP ratio stands around 40-60%. The long-term debt ratings are mostly AAAs, with a few exceptions: Belgium (AA+), Greece (A), Italy (AA) and Portugal (AA-).

[22] Some additional but partial information can be found in the paper of Hirotaka Inoue, The Structure of Government Securities Markets in G10 Countries:Summary of Questionnaire Results,Financial Markets Department,Bank of Japan

Table 2: Outstanding Debt & Ratings

	Outstanding debt				Long-term debt rating		
	EUR, bln		% of GDP[23]				
	short term[24]	long term[25]	short term[2]	long term[3]	S&P	Moody's	Fitch
Austria	1.2	124.6	0.5	51.6	AAA	Aaa	AAA
Belgium	29.9	233.9	10.3	80.8	AA+	Aa1	AA
Finland	2.3	48.8	1.5	32.0	AAA	Aaa	AAA
France	94.1	830.9	5.6	49.5	AAA	Aaa	AAA
Germany	35.9	1018.1	1.6	45.6	AAA	Aaa	AAA
Greece	1.6	170.1	0.9	98.7	A	A1	A
Ireland	0	31.4	0.0	20.4	AAA	Aaa	AAA
Italy	139.2	1076	10.1	78.3	AA	Aa2	AA
Luxembourg	0	0.2	0.0	0.8	AAA	Aaa	AAA
Netherlands	19.5	199.8	4.0	40.5	AAA	Aaa	AAA
Portugal	12.7	67.4	8.8	47.0	AA-	Aa2	AA
Spain	38.3	295	4.4	34.1	AAA	Aaa	AAA

Source: ECB (July 2005); Standard & Poor's, Moody's, Fitch (October 2005).

Overall, the average life of government debt is 4-6 years, with maturity spectrums of 3 to 18 months (treasury bills) and 2-30 years (bonds). The dominant bond maturity is 10 years.

In the early years of the European Monetary Union, lower budget deficits led to a reduction in the net borrowing requirements. In the following years, due to lower economic growth, there was a shift from a budget surplus for the euro area of 0.1% of GDP in 2000 to a deficit of 2.7% of GDP in 2004. As a consequence, the gross borrowing requirements of euro area countries increased. For example, the net borrowing of the biggest debt issuers, Italy, Germany and France, was as high as the volume outstanding of the seven "small" euro area countries together.

Table 3: Composition of Debt, 31 December 2003

	% euro	Avg Life	Duration	Avg Yield
Austria	n/a	n/a	n/a	n/a
Belgium	99	5.9	3.9*	5.1
Finland	77	3.9	2.3	4.1
France	100	5.6	n/a	n./a
Germany	100	6.3	n/a	n/a
Greece	99	6.2	3.7	5.4
Ireland	100	6.6	4.9	4.1
Italy	98	6.1	3.6	2.7
Luxembourg	n/a	n/a	n/a	n/a
Netherlands	100	6.0	3.8	n/a
Portugal	100	4.3	2.9	6.3

[23] At market prices.
[24] Under 1 year.
[25] Over 1 year.
* Euro-denominated debt only.

Spain	97	6.2	4.7	n/a

The figure below illustrates the shifts in the relative debt levels over the last few years. The market environment, with historically low yields, provided convenient conditions for issuing long-term debt. In addition, the high quality of euro area sovereign bonds furthers their acceptance in repo transactions or in open market operations and lending facilities. As of July 2005, the share of short term debt (maturities under 1 year) remained under 5% in most countries. The exceptions are Belgium (10.3%), France (5.6%), Italy (10.1%) and Portugal (8.8%).

Figure 17: Relative Debt Levels of Eurozone Governments

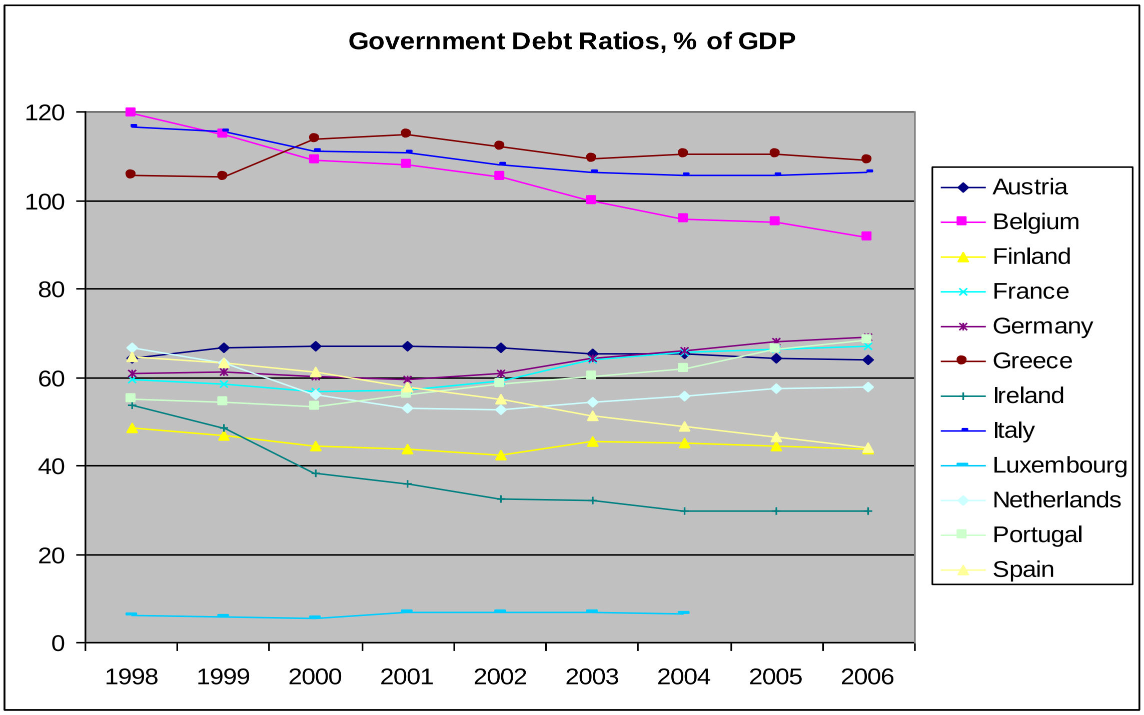
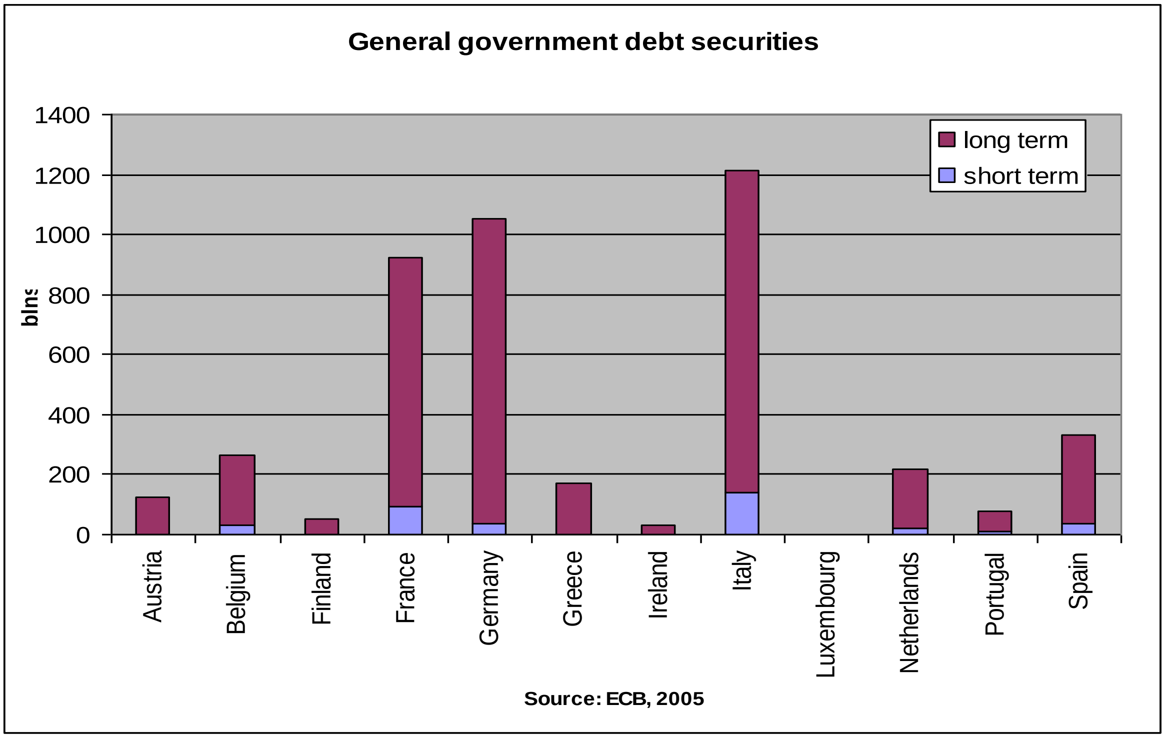


Table 3: Composition of Debt, 31 December 2003

	% euro	Avg Life	Duration	Avg Yield
Austria	n/a	n/a	n/a	n/a
Belgium	99	5.9	3.9*	5.1
Finland	77	3.9	2.3	4.1
France	100	5.6	n/a	n./a
Germany	100	6.3	n/a	n/a
Greece	99	6.2	3.7	5.4
Ireland	100	6.6	4.9	4.1
Italy	98	6.1	3.6	2.7
Luxembourg	n/a	n/a	n/a	n/a
Netherlands	100	6.0	3.8	n/a
Portugal	100	4.3	2.9	6.3
Spain	97	6.2	4.7	n/a

Source: MTS Group (March 2005).

Figure 18: Government Debt Maturity Profile



Source: ECB (July 2005).

Debt Instruments

The prevailing debt instrument of Eurozone Governments is plain vanilla fixed rate bond auctioned in fungible tranches to a specific group of primary dealers. Such instruments typically account for 80-90% of total debt. The index-linked bonds are the French OATi and OAT€i (Obligations Assimilable du Tresor linked to the national consumer price index and the Eurozone consumer price index, respectively) that account 8.5% of the total French government debt, and Italian BTP€i (Treasury Bonds linked to the Eurozone CPI) that account for less then 1% of the total Italian government debt. The government debt breakdown by the type of debt instrument is presented in Table 4.

Another notable exception is Austria, where standardized issuance programs were developed to stimulate foreign borrowing. One program is EMTN (European Medium Term Notes), which a program for issuing foreign currency bonds, index-linked bonds and notes and other non-standard instruments on foreign securities market under English Law. The other program is Australian Dollar Medium Term Notes that was developed to tap on the Australian securities market. No information was available on the volumes of issuance under these programs.

Government bond repos exist on all Eurozone markets.

Table 4: Eurozone Government Debt Instruments

Country	Instruments	Share of total debt	Maturities
Austria	Government EUR-bonds	data not available	up to 50 years
	EMTN securities[26]		n/a
	Australian Dollar Medium Term Notes[27]		n/a
	Austrian Treasury Bills		up to 1 year
Belgium	Traditional loans	2.18%	n/a
	OLOs	78.00%	up to 30 years
	State notes	3.24%	n/a
	Treasury certificates	9.95%	up to 1 year
Finland	RFGB bonds	80.00%	up to 10 years
	T-bills	9.00%	up to 1 year
	Foreign Currency Debt	2.00%	n/a
	Other	9.00%	n/a
France	OATs (bonds); OATi[28], OAT€i[8]	65.00%	7 - 30 years
	BTAN (notes)	21.00%	2 - 5 years
	BTF (bills)	14.00%	12 - 52 weeks
Germany	Bunds (federal bonds)	data not available	10, 30 years
	Bobls (federal notes)		5 years
	Schatz (federal notes)		2 years
	Bubills (treasury discount paper)		6 month
Greece	GGB - Greek Government Bonds	85.80%	3, 5, 10 and 20 years
	T-Bills	0.90%	3, 6, and 12 month
	Bank of Greece Loans	4.40%	n/a
	Other Loans	8.90%	n/a
Ireland	EUR-denominated bonds	83.00%	up to 15 years
	Commercial paper & T-bills	1.00%	n/a
	Retail Savings Instruments	12.00%	n/a
	Other	4.00%	n/a
Italy	BTP (fixed rate)	59.68%	3, 5, 10, 30 years
	BTP€i[8]	0.88%	3, 5, 10, 30 years
	CCT (floating rate)	17.07%	7 years
	BOT (T-bills)	10.34%	90, 180, 360 days
	CTZ (zero coupon bonds)	4.55%	18, 24 month
Luxembourg	Treasury Bonds	100%	10 years
Netherlands	DSL (Dutch State Loans)	91.70%	3, 5, 10 and 30 years
	DTC (Dutch Treasury Certificates)	8.30%	3, 6, 9, 12 month
Portugal	OT (Obligacoes do Tresoro)	62.00%	1 - 30 years
	BT (Bilhetes do Tresoro)	11.00%	3, 6, 12 month
	Other bonds	4.00%	n/a
	Other non-negotiable debt	23.00%	n/a
Spain	Obligaciones de Estado	80.20%	3, 5 years
	Bonos de Estado		10, 15, 30 years

[26] Foreign currency bonds, inflation-linked bonds, (callable) fixed rate/index linked notes etc. issued under English Law.
[27] Issued under Australian Law.
[28] Inflation-linked.

	Letras de Tresoro	11.60%	3, 6, 12, 18 month
	Other debt	8.20%	n/a

Source: MTS Group (March 2005).

One clear trend in the government bond market is towards larger issue sizes as national debt managers have focused on improving liquidity in their instruments by launching benchmark bonds of €5 billion or more in order to be eligible for trading on the EuroMTS electronic trading platform. Bonds up to a volume outstanding of € 20 billion have an 80% share of total bonds outstanding, while small bonds of up to €500 million have all but disappeared and bonds up to €5 billion have a market share of only 4% of bonds outstanding. Smaller countries in particular are tending to increase the volume of existing bonds rather than issue new debt to obtain market liquidity (fungible issues). Certain countries have also arranged programs to buy back or exchange bonds primarily in order to increase the liquidity of on-the-run issues and to exchange old illiquid bonds.

The maturity spectrum up to ten years is relatively homogenous in terms of the share of sovereign issuers. More interesting in terms of fragmentation is the long-term spectrum, where no single maturity is offered by all of the six biggest euro area sovereign issuers. These incomplete maturity structures for each issuer result from the different sizes of the debt requirements of the countries concerned. Only big countries are able to serve the whole maturity spectrum.

The coupon structure itself did not change significantly over the last decade; however, there is an evident tendency towards issuing index-linked bonds in order to decrease the costs of debt financing.

Debt Issuance and Trading

Harmonization in public debt management is observable, as individual debt managers endeavor to achieve greater transparency in their strategies and to develop standardized tools for issuing debt.

One trend is towards greater transparency, with information about public debt being published on the internet and in periodical bulletins and annual reports. One example is the standardization in the regularly published and pre-announced issuance calendar. Every country has a calendar which contains the date and the planned issue volume. However, the calendars vary in terms of the time horizon. Some plan for the next three months and others for the next whole year. In addition, the binding character of the issue calendar differs with regard to the time as well as to the issuance volume.

For the primary market procedure nearly all countries' issuing activities generally use the traditional auction method. However, a method combining an auction procedure with the use of syndication is also becoming more common. The secondary market activity traditionally takes place in the wholesale over-the-counter (OTC) market. In the euro area the widely recognized need to create the facilities necessary for cross-border trading gave rise to one technological innovation. Designed by the Italian MTS Group, the London-based EuroMTS was set up as an electronic trading system for euro benchmark bonds with a pool of instruments including government bonds of every eligible issue within the EU. On average, MTS accounts for around 65% of the Eurozone government debt instrument turnover. Government bonds are also listed on the local stock exchanges, bu the turnover there is minimal.

Another trend in the trading environment is the virtually exclusive use of primary dealer systems as competition between issuing Member States has intensified and the national governments need to broaden their investor base throughout Europe and beyond. Therefore nearly all euro area countries have implemented a primary dealer system. The primary dealers are obliged not only to buy securities when issued, but also to set ask and bid prices in the secondary market. In order, in particular, to increase the attraction for international investors, the extension of the working relationship to foreign primary dealers increased. The emergence of an international corps of primary dealer banks present in a majority of EU sovereign issuers is an interesting phenomenon.

Table 5: Market Infrastructure (Bonds)

	Issuing	**Trading**	**Settlement**
Austria	Auctions about every 6 weeks, irregular syndicated issues	Vienna Stock Exchange, Paris and Frankfurt Stock Exchanges; MTS Austrian Market, EuroMTS*	Oesterreiche Kontrollbank AG; Euroclear, Clearstream, LCH.Clearnet
Belgium	Auctions every other month	Brussels Stock Exchange, MTS Belgium, EuroMTS*	National Bank of Belgium, LCH.Clearnet
Finland	Single price auctions for primary dealers, syndications is possible for new benchmark issues	Helsinki Stock Exchange, MTS Finland, EuroMTS*	Euroclear, ClearStream
France	Regular auctions announced in advance, syndication is possible for specific products	Euronext Paris, MTS France, EuroMTS*	Euroclear France, LCH.Clearnet
Germany	Regular auctions with voluntary participation (no primary dealers)	German Stock Exchanges, EuroMTS*, MTS Deutschland, Eurex Bonds	Euroclear, ClearStream, LCH.Clearnet
Greece	Syndications, public auctions, taps and private placements	Athens Stock Exchange, HDAT, MTS Greece, EuroMTS*	Bank of Greece
Ireland	Monthly auctions (February through November), exclusively for primary dealers	Irish Stock Exchange , MTS Ireland, EuroMTS*	Euroclear
Italy	Monthly and quarterly single price auctions	MTS Italy, EuroMTS*, major European stock exchanges	Express II, LCH.Clearnet
Luxembourg	n/a	n/a	n/a
Netherlands	Regular tap auctions for primary dealers, Dutch Direct Auctions for end users (bids placed through primary dealers)	Amsterdam Stock Exchange, MTS Amsterdam, EuroMTS*	Euroclear, ClearStream, LCH.Clearnet

This version of paper is prepared by Sergey Smirnov, EBC member; Alexey Zakharov; Roman Rachkov; Victor Lapshin; Vladimir Zdorovenin; Stepan Evstratov

Portugal	Monthly multi-price auctions for primary dealers (competitive and non-competitive)	MEDIP/MTS Portugal, EuroMTS*	Euroclear, Clearstream
Spain	Monthly modified single price auctions	OTC market, MTS Spain, Senaf, Madrid Stock Exchange, EuroMTS*	Iberclear

* - Benchmarking issues over €5 bln.
Source: MTS Group (March 2005).

Table 6: Market Participants (Bonds)

	Primary Dealers	MTS market makers	International Investors, %
Austria	25	21	90-99
Belgium	16	19	49
Finland	14	21	77
France	21	22	42
Germany	40[29]	30 (Bunds)	n/a
Greece	21	21	55
Ireland	8	10	79
Italy	22	31	47
Luxembourg	n/a	n/a	n/a
Netherlands	13	13	81
Portugal	15	16	85
Spain	20	18	38

Source: MTS Group (March 2005).

Another sign of harmonization of the Eurozone debt markets is the adoption of universal conventions in regards to interest and yield calculation. It should also be added that Eurozone governments are taking effort towards coordination of the coupon payments on the new issues. The payment conventions currently in force on the Eurozone debt markets are presented on Table 7. As it can be seen, the accrued interest is calculated on Actual / Actual convention. Interest on coupon bonds is paid annually, with the exception of Italy, where coupon is paid twice a year. Yields are calculated based on the ISMA convention, with the exception of Italy where the yields are calculated based on annual convention.

Table 7: Settlement/Payment Conventions (Bonds)

	Settlement Date	Interest Accrues	Interest Paid	Yield Method
Austria	T + 3	Actual / Actual	Annual	Annual
Belgium	T + 3	Actual / Actual	Annual	ISMA
Finland	T + 3	Actual / Actual	Annual	ISMA
France	T + 3 international; T + 1 domestic	Actual / Actual	Annual	ISMA
Germany	T + 3	Actual / Actual	Annual	ISMA
Greece	T + 3	Actual / Actual	Annual	ISMA
Ireland	T + 3	Actual / Actual	Annual	ISMA
Italy	T + 3	Actual / Actual	Semi-annual	Annual
Luxembourg	n/a	n/a	n/a	n/a
Netherlands	T + 3	Actual / Actual	Annual	ISMA

[29] Market participants, no primary dealers.

Portugal	T + 3	Actual / Actual	Annual	ISMA
Spain	T + 3	Actual / Actual	Annual	ISMA

Source: MTS Group (March 2005).

Tightening Benchmark Government Bond Yield Spreads

Elimination of the foreign exchange risks with the introduction of Euro and intensive harmonization of the Eurozone debt markets resulted in a notable tightening of the government bond yield spreads. Currently, the yields mainly result from the differences in credit quality of the obligors and the market liquidity. Unfortunately, we were unable estimate and compare the liquidity of the Eurozone debt market given the available data.

Other issues, such as taxation effects and market structure, can have an impact on credit spreads and risk-free yield curves. However, government bonds are mainly traded by financial institutions that are not subject to withholding taxation of interest income, which means that taxation aspects should not have substantial impact on the credit spreads. However, such issues as existence of widespread futures or derivative contracts based on particular government debt instrument may impact the demand for these securities and resultantly the credit spreads. This aspect of the Eurozone debt market is left out of the scope of this review.

Table 8: Harmonized spreads of euro area countries' long-term government bond yields against Germany.

	Spreads (in b.p.)			Changes in spreads		
	Jan'99	Jan'01	Dec'03	Jan'99 - Dec'03	Jan'99- Jan'01	Jan'01 - Dec'03
Austria	14	30	11	-3	16	-19
Belgium	20	36	9	-11	16	-27
Spain	18	28	5	-13	10	-23
Finland	21	22	4	-17	1	-18
France	7	14	5	-2	7	-9
Ireland	19	23	7	-12	4	-16
Italy	22	38	17	-5	16	-21
Netherlands	10	13	4	-6	3	-9
Portugal	20	36	11	-9	16	-25
Average	16.78	26.67	8.11	-8.67	9.89	-18.56
Greece (Since 2001)		55	16	-		-39
Average (including Greece)		29.5	8.9	-		-20.6

Source: ECB, 2004 (based on Eurostat and Deutsche Bank calculations)