**Universal method of valuing inflation-linked bonds.**

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# Stripping of indexed OAT

Latest developments in financial modelling have made possible to develop and value products calibrated as closely as possible to the expectations of the various investors on all asset classes and on the entire range of financial activity. This race for innovation has mainly focused on the development of ever more sophisticated products, but we can still mention some movements to simplify traditional financial products into elementary components. The aim is, of course, to reassemble these unitary elements in the best interests of investors.

Bond stripping is one of these "simplifying" innovations, it consists in allowing the division of the sequence of flows composing an ordinary bond into a series of flows negotiable independently (stripping) and vice versa (consolidation). The accounting procedure is relatively simple, it simply consists of crediting the book custodian with the securities created and debiting him or her with the securitie(s) necessary for the transformation. The arbitrated valuation of securities entering the process: bonds, coupons and principal flows, on the other hand, can be a problem especially when the elementary flows from several different bonds become fungible after dismemberment as is the case on OAT (Assimilable Treasury Bonds).

This problem has long since been solved by all SVTs (Specialists in Treasury Securities) for fixed-rate bonds (authorized for dismemberment in 1991) but arouses less interest for inflation-linked bonds(althoughauthorized fordismemberment since April 2007). On the one hand, there is a risk that strips (dismembered bonds) of real rates may accumulate the illiquidity of fixed rate strips and real rate bonds, which would be not encouraging for the institutions responsible forensuring market liquidity; on the other hand,it is likely to highlight an increase in pricing complexity thatis not conducive to the democratization of the product. The following article does not propose an answer to liquidity problems but develops a simple method of pricing dismembered bonds which can easily be implemented by the institutions interested in this product.

At that date there are no reference quotations for the real rates that can be consulted by the usual broadcasters of stock market information. Given current levels of rates, it is possible that there will not be rates for a long time. The interest of the approach proposed here lies rather in its universality: it makes it possible to determine the arbitrated price of real zero coupons and by extension of any linear combination of real zero coupons by integrating the levels of nominal rates, inflation and credit/liquidity specific to a given issuer. It therefore makes it possible to value all bonds indexed toinflation.

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## Fungibility of zero-coupons and adjusted amounts

The adjustment of dismembered amounts is an essential mechanism to ensure the fungibility of zero coupons given the method of calculating inflation-linked bonds.

This mechanism has no equivalent for nominal bonds:dismembering €100 million of bonds paying a fixed coupon of x% is equivalent to replacing a line of €100 million of the initial security with a line of €100 million + x M€ of a zero coupon of the same maturity as the initial security (we no longer distinguish the zero-coupons from the coupon flows from those from the principal flow) and n-1 lines of x M€ of zero-coupons of the same maturities as the n-1 first coupons of the initial security. Nominal and coupon rate data are sufficient to calculate the amounts of securities to be created/replenished and no adjustment is required.

The way inflation-linked bonds are calculated ensures that the nominal value of the security moves in line with inflationindices. The real value of the securities (in constant currency) is simply multipliede by the performance of the inflation index between the date of issue and the date of transaction. The nomina being linked to a performance it follows that the operations of dismemberment / consolidation will they be themselves conditioned by the performance of inflation.

#### Remarks

The index used is usually a formatted version (contractual setting) of the raw published index. We will not detail here all the types of formatting of gross inflation indices to make them indices applicable to transactions, but we can briefly mention the main ones:

* Addition of a time lag (lag): essential given the time required for economic organizations to calculate and publish the indices corresponding to an observation period (usually a calendar month). The observed lags are usually two or three months away.
* Index smoothing: transactions are daily but the indices are published at a lower frequency (monthly or semi-monthly) and show a discontinuity with each publication. The indices used to value transactions can therefore be smoothed (linear interpolation) to avoid valuation jumps. It is noted that this type of smoothing is sometimes present on bonds (e.g. RQI: daily inflation ratio for OATi) but more rarely on vanilla derivatives.
* Seasonal adjustment: inflation indices can be eminently seasonal, there are regular fluctuations over the years corresponding to recurring one-off economic events (balances ...). Treatments aimed at spreading inflation over the calendar year are sometimes implemented on derivatives but remain exceptional on bonds.

In order to maintain the fluidity of the presentation, thetreatments actually applied to inflation indices should not bedistinguished and the formatted index applicable to flows traded on date t should be noted generically in the cpi(t) (Preconditioned Index for Calculations).

The performance of inflation between the date of issue e0 and t is written .

The nominalflated corresponding to K M€ of securities issued on date e0 is therefore equal to (M€) while the nominalflated corresponding to K M€ of securities issued on date t1 is equal to (M€).

The amounts of zero coupons created during a dismemberment must of course reflect theobservable difference between the inflated nominals without, however, creating a difference between the zero coupons resulting from dismemberments carried out on different dates: CPI(t) vs CPI(t'). It is therefore necessary to hide the impact of initial inflations without integrating current inflation.

The solution appears by rewriting the inflated nominal as .

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| We call *nominal adjusted base 100*, lavalue  It corresponds to the unflated nominal of an inflation-linked security with an inflation base equal to 100. |

#### Notes

* If the zero coupons resulting from the dismemberment of the basic title CPI(e0) are to be rebonded into the basic title CPI(e1), the nominal reembré is deducted from the equality of the adjusted nominals: . Either, in nominal amounts not inflated:

The new basis for calculating these zero-coupons after consolidation is obviously that of the reconstituted security, i.e. CPI(t1).

* Base changes are made periodically by the bodies in charge of measuring inflation in order to bring current indices back to levels close to 100. The adjusted nominals are completely dependent on the base level of the price indices, so they must be recalculated after each rebasing of the index.

If the initial CPI(e0) levelis changed to CPI'(e0) in the new base, the adjusted nominal becomes:

* The face rates of indexed bonds being generally low, i.e. 0.1% on recently issued OATi, the coupon amounts obtained by dismemberment are themselves extremely low, i.e. €100k for €100M in the case of a coupon of 0.10%, which poses a real liquidity problem for indexed strips outside the repayment maturities.

### Conclusion

By analogy with nominal securities, we can write for inflation-linked bonds:

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| Dismembering K M€ (nominal at issue) of basic indexed bonds CPI(e0) and coupon x% amounts to replacing a line of K M€ of the initial security with a line of K M€ of a zero coupon of the same maturity as the initial security and n lines of M€ of zero-coupons of the same maturities as the coupons of the initial security.  The base of the zero coupons created is equal to 100 and  It should be noted that in the case of indexed OAT, principal and coupons are not fungible because of the option of capital protection carried only by the principal. |

## Pricing of indexed zero-coupons

The amounts of securities created are known, it remains to determine the trading values. Several approaches are possible: *standalone*creditcurve, asset-swaps, Z-spread, ... Although the Z-spread is a little less direct (implicit data), we will retain this method sufficiently precise for our spot pricing needs and extendable by more sophisticated credit models if necessary; if we neglect the aspects related to the liquidity (cash and repo) of the different assets, a Z-spread is comparable to a CDS recovery spread 0%.

### Reminder: theoretical stripping of a nominal bond

There are two curves

* A discount curve deemed risk-free (€ster, ois, ...) which at any maturity combines a risk-free rate or a discount factor (present value of a monetary unit paid on date T).
* A curve of unit Z-spreads which for a given issuer associates a credit spreadwith any maturity flow t and Θ0 les parameters of this curve.

By construction, the following relationship must be verified for each obligation *O*j of the calibration set:

Where is the market price (coupon attached) expressed as a percentage, the nominal rate, the number of coupons attached and the coupon deadlines.

We deduce the value of a zero-coupon of maturity t of the same issuer:

#### Remarks

* The parameters Θ0 defining the Z-spread curve are almost always deduceds from the system of equations resulting from the application of theequilibrium relation to each obligation of the calibration set.
* The shape of the Z-spreads curve is not constrained, the simplest is discontinuous (constant by pieces) but we can also choose it continuous (affine by pieces) seee continuouslyderivable (splines, functionals ...) ... depending on the use made of it.
* The number of parameters may exceed the number of available relationships (the number of titles of the calibration set) and additional assumptions may be necessary: i.e. exogenous determination of the slope of the first segment for an affine curve by pieces ...
* When the number of parameters is less than the number of available relationships, the classical calculation of the Lagrangian makes it possible to determine the credit curve minimizing the distance to the calibration set.
* Given the theoretical nature of the zero-coupon prices sought (the negotiated prices differ from the theoretical prices in particular because of the liquidity disparities between zero-coupons and conventional bonds), the relative instability of Z-spreads and the intended use(spot prices, no forwards or diffusion),a constant curve in pieces based on bond prices is generally sufficient.
* Taking into account convexity adjustments applicable to zero maturity coupons not present in the calibration set would require modelling the diffusion of theZ-Spread, whichwould be beyond the scope of this study.

### Stripping of inflation-linked bonds

The two curves used for stripping nominal bonds are no longer sufficient and an inflation curve is added to them:

We do not go into the details of the construction of such a curve which, in increasing degrees of refinement, will be based on:

* Zero-coupon inflation swap quotes (annual) - minimum
* A seasonality model whose calibration is most often based on calendar (monthly) spread quotes and past data – necessary when seasonality is marked
* Convexity adjustments applicable to maturities not equal to those of zero-coupon swaps – not necessarily material when the benchmark CPI is known (standard for indexed bonds)

It is simply assumed that the valuation tool incorporates all the conventions and practices specific to the inflation market under consideration and is able to provide the appropriate level of CPI for the securities to be dismembered. Nor is there a distinction between deterministic CPIs (indices based on previously published inflations) and CPI quoted on the market.

In the case of indexed bonds, with the same ratings as for nominal bonds, the equilibrium relationship of nominal amounts becomes taking into account the different inflation bases between bonds and zero-coupons:

By noticing that then factoring by the CPI ratio at the date of transaction, we deduce the real value of an indexed zero-coupon of maturity t:

This value corresponds to the expected performance of inflation over the remaining life of the security discounted in accordance with the issuer's credit.

The nominal value of the zero coupons rebased at 100 being equal to:

For reasons of homogeneity with the inflation market as a whole, it seems appropriate to quote in accordance with the practices of the market in question (i.e. in real value in Europe and in nominal value in Great Britain).

#### Remarks

* For reasons related to the differences in liquidity between nominal and indexed bonds, it seems inappropriate to postpone the Z-spread levels calculated for nominal bonds to indexed bonds.
* If modelled, convexity adjustments applicable to Z-spreads would have to include not only impacts due to credit/nominal rate correlations but also credit/inflation correlations.
* All the remarks already made for nominal bonds are applicable to indexed bonds.

## Example

An example has been fully calculated to illustrate the approach. It is based on inflation-linked bonds issued by the French Treasury that are actually eligible for dismemberment. The data used for the example was collected on 30/07/2021.

The results available below remain illustrative of an approach and the associated spreadsheet cannot be used in the state as a support for transactions (i.e. the calendar of public holidays is limited to weekends,seasonality has simply been deduced from historical CPIs without calibration on market data, ...).

### Discount curve

The discount curve has been deducted from the €strswaprates. Completely collateralized,it is comparable to a risk-free curve. Thispoint is not essential and the same calculations could be made based for example on Euribors swaps, the credit/liquidity spreads would be of another magnitude but the rates of zero-coupons indexed to inflation would be very close to those obtained here.

Discount factors are deducted from the two segments of quoted curves: the short-term segment consisting of maturities from zero to two years and the long-term segment for maturities from two years to fifty years.

The rates of intermediate maturities (i.e. from 16 years to 19 years) are deduced by linear interpolations on the t to theknown (rated).



In the attached Excel sheet, this data is used through the calculation functions

* DiscountRate (Maturity, Curve, Curve)
* DiscountFactor(Maturity, Curve, Curve)

### Inflation curves

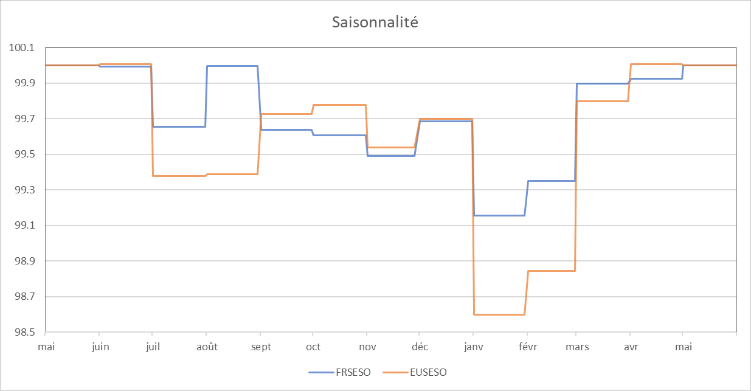
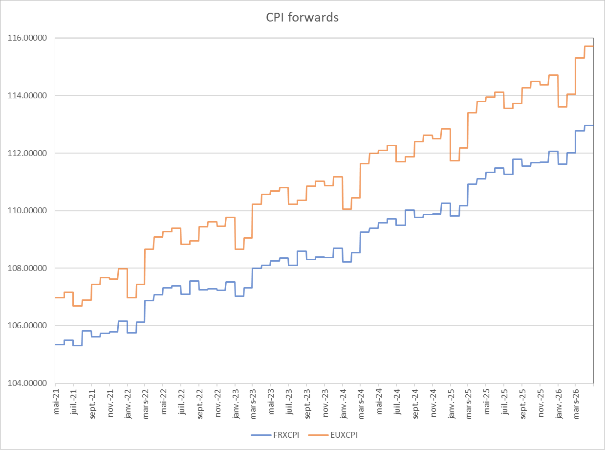
Annual CPI forwards are deducted from zero-coupon swap rates and their interpolations.



Taking into account seasonality then makes it possible to determine the monthly CPI forwards. Seasonality is a relatively stable periodic shift observable on inflation: i.e. when inflation is worth 1.2% / year, monthly inflation is not equal to 0.1% but to 0.1% increased by the seasonality coefficient of the current month.

Seasonality vectors have been deduced from historical CPI series observed since 2011 but have not been recalibrated from specific market quotes (i.e. Reset ICAP).



### Credit spreads

As part of this presentation, the Z-spreads of inflation-linked OAT serve as a support for estimating a credit/liquidity curve.

As a reminder, for a givendiscount curve andbond, the Z-spread is the offset (translation) of the discount curve that achieves the equality of the market price and the present value of the bond flows, this spread is sometimes assimilated to a spread of CDS 0-recovery. In practice, the Z-spread combines a credit component common to all the securities of the same issuer and a liquidity componentspecific to eachsecurity. In this presentation, the calibration panel is reduced to indexed bonds, it is sufficient to ensure the pricing of indexed zero-coupons but should be complemented by nominal securities to precisely calibrate a credit curve valid for any liquidity class – indexed bonds being relatively illiquid compared to nominal bonds.

A Z-spread is specific to the bond to which it is attached or zero-coupons of the same maturity resulting from dismemberments of different bonds are fungible and therefore indistinguishable, so we adapt the modeling by considering no longer a set of independent Z-spreads but a curve of Z-spreads: the offsets of the discount curve are no longer applied to the flows of a given security but to all flows of the same maturity. This approach is of course not applicable to securities from different issuers or different liquidity classes. Taking into account the calculated Z-spreads (see table), we distinguish the Z-spread curve of OATei from that of OATi. The latter seem much more expensive relative to swap curves.

Functionally, the Z-spread curves considered are constant in pieces, the jumps corresponding to the maturities of the OAT.



Note: as mentioned earlier, the indexed OAT repayment flow has a capital protection clause whose value is not zero - but tends to decrease rapidly when inflation remains positive. To be perfectly precise, the forward value of the floor 0% implicit in the protection of capital must be added to the forward value of the repayment flow prior to the evaluation of the credit/liquidity spread because this option is specific to the terminal flow. In the absence of pricing elements (cap/floor volatility table),thecalculation presented does not include this element and therefore displays a slightly better issuer credit than the credit calculated floor included.

### Prices of ZCs

By assembling the different elements: discount, credit/liquidity and inflation, we deduce the prices of zero coupons indexed to inflation.

Prices and rates are given in real terms in accordance with the usage on OAT. As an indication, we have added the nominal prices on the zero-coupons of the July maturities indexed to European inflation.



Taking into account the credit spread we can check the real rates calculated by estimating them by Fischer relation:

Or even its development in the first order:

On the French inflation coupon of 25/07/2022 the development in the first order gives a real annual rate of the order of -2.954% (: -0.585%: - 0.574%and 1.644% seasonality included) which confirms the plausibility of the calculated rate: -2.959%.

Similarly, on the European inflation coupon of 25/07/2047: from the Fisher relationship we expect an annual real rate close to -1.184%(: 0.168%: 0.491% and 1. 865%seasonality included) which validates thecalculated score: -1.172%.

The quality of the exact calculation or Fisher approximations obviously depends on the quality of the Z-Spread calculation. As credit/liquidity spreads are not published on inflation-linked OATs, it is difficult to avoid this step in the calculation. On the other hand, the manipulation of the discount and inflation curves are perfectly controlled by market participants and do not require specific developments.