The French Bond Association
Comité de Normalisation Obligataire

CURRENT STANDARDS ON FIXED INCOME

PRICING FIXED INCOME SECURITIES AGAINST AN INTEREST SWAP CURVE

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Summary

Pricing fixed income securities against interest rate swaps

Introduction to volume 2

This second volume proposes a different valuation of fixed income securities compared with the one described in the first volume:

- **The first volume** describes the structure of bonds and other fixed income securities, the calculation of their cash flows and their yield formulae based on pure actuarial conventions. According to these conventions, bonds' future cash flows are priced by using a flat yield curve corresponding to their own yield to maturity, without making any difference between bonds priced at par and bonds bearing a price different from par.

  This pure actuarial yield to maturity formula is very simple to implement and offers a robust estimation of the actuarial modified duration and convexity of a given bond. However, this yield refers only to the specific bond that is valorized and cannot compare accurately the relative value of bonds bearing different market prices.

- **This second volume** refers to the same general market conventions as the one described in the first volume for the calculation of the cash flows of fixed income instruments in euro. However, it uses a different approach for the valuation of fixed income instruments, pricing them against the zero coupon curve derived from the par interest swap curve. This more sophisticated methodology based on a common benchmark reference curve, the interest rate par swap curve, allows for a better comparison between the yields of various fixed income instruments launched on a same maturity by different issuers that bear different coupons and different market prices.

This volume concentrates on the valuation of plain vanilla fixed income securities based on the interest rate par swap curve, excluding in principle the pricing of exotic bonds.

- It aims to explain the main concepts used for the interest rate derivative valuation of fixed income securities:

  1) How the actuarial pricing methodology of plain vanilla fixed income instruments differs from the one based on a derivative valuation,

  2) The rate relationship between the par swap curve, the zero coupon curve and the forward curve that are derived from this par swap curve.

  3) And describes the valuation of some plain vanilla fixed income securities, notably fixed rate bonds, I/L bonds and floating rate bonds referenced to a long term interest rate reference like CMS (Constant Maturity Swaps) or CMT (Constant Maturity Treasuries) bonds.

- It does not intend to detail extensively each specific structure of Over the Counter, OTC, or Exchange Traded Derivatives, and the exotic structures of some fixed income securities.

The valuation of plain vanilla interest swaps has been strongly impacted, since August 2007, by the current financial turmoil and we expose the recent evolution of plain vanilla derivative pricing.

1. Interest Rate Swaps: the benchmark reference used to compare different bonds' yields

1.1 Before 1999, new fixed income issues were mainly priced against a sovereign curve

In FRF for instance, the spread used to price a new fixed rate bond was generally expressed as the difference between its yield to maturity and the yield to maturity of an OAT bearing a similar or very close maturity. A 10 year corporate[1] new issue launched in 1998 maturing in May 2008 could have been priced:

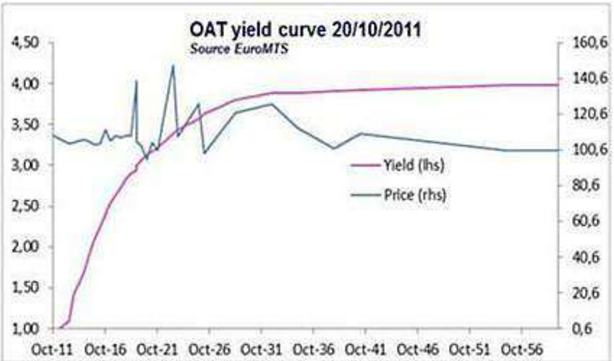
- Either by reference to the yield of the 5.25% 25 April 2008 OAT,

- Or by reference to the interpolated OAT curve calculated between the 5.25% 25 April 2008 OAT and the 8.5% 25 October 2008 OAT.

The price of a new issue is usually set at or around par, and consequently its coupons, are more or less equal to its yield to maturity. In such a case, the yield to maturity formula can be considered as pertinent to compare this yield with a benchmark reference like a sovereign curve.

However, the benchmark sovereign curve is far from being a par curve: This is due to the policy of sovereign issuers that tend to limit the number of their bonds[2], in order to increase these bonds' liquidity. Sovereign borrowers also issue long dated bonds, up to 50 years, and some of their old still outstanding issues pay a coupon that is far away from current market yields.

The right hand scale of the 1st chart shown below, illustrates the price of the various OATs included in the French yield curve. Some of these OATs have been launched before the introduction of the € and they are priced well above par: On 20 October 2011, the 8.5% 25/04/2023 OAT priced at 148.03% had a life to maturity of 11.5 years and a duration of 8.2 years. By comparison, the 3.25% 25/10/2021 OAT had a 10 year life to maturity, shorter by 1.5 years than the 8.5 25/04/2023 OAT, however its 8.7 year duration was 6 months longer.



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[1] Unless explicitly mentioned, this 2nd Volume qualifies as "corporate bonds", bonds launched by non-sovereign issuers.

[2] For example, 10 year French OATs are usually launched every 6 months and they are regularly reopened, so as to offer investors more liquid issues that benefit from their large outstanding amount.

Using as a benchmark reference a bond priced far away from par can be misleading:

- As previously explained, the market yield to maturity of a same issuer and same maturity fixed rate bond depends on its nominal coupon and consequently on its market price[3]. But the sole yield to maturity formula based on a flat yield cannot give a theoretical valuation of this price differential[4].

- In this 2nd volume we indicate how, by using the zero coupon curve derived from the par swap rates, we can price the theoretical yield differential between same maturity bonds bearing different coupons and consequently different market prices.

1.2 Since January 1999, new issues are usually priced by reference to the Euribor swap curve

The Euribor interest rate par swap curve, a curve bearing a coupon that is equal to its yield to maturity, is nowadays generally used as the common reference for the relative pricing of new corporate bonds:

- Contrary to a fixed rate bond curve made of secondary issues bearing different prices, the swap curve is a real time updated primary par curve,

- And the yield to maturity of this par curve is homogeneous with the yield to maturity of a new issue priced at or around par.

2. Interest Rate Derivative markets are by far the largest fixed income markets in the world

One of the main reasons explaining the importance of IRS seems to be linked to the yield curve shape: On average, an interest rate curve, either a bond or a derivative one, should be normally positively slopped, due to the long term risk aversion of institutional and retail investors.

In a steep curve environment, fixed income investors seeking to secure high level of yields, tend to buy long dated corporate securities. But corporate issuers , most of the time tend to swap their fixed rate exposure against a floating interest one, in order to reduce their immediate cost of financing and to match their ALM requirements; corporate borrowers' turnover depends generally on short term economic conditions.

These four charts following illustrate the September 2010 BIS foreign exchange and derivative survey[5], based on April 2010 BIS statistics:

- **Average daily turnover:** the overall OTC daily IRS markets' turnover amounted to around USD 2.057 billion. The 1st chart details this daily turnover by types of instruments.

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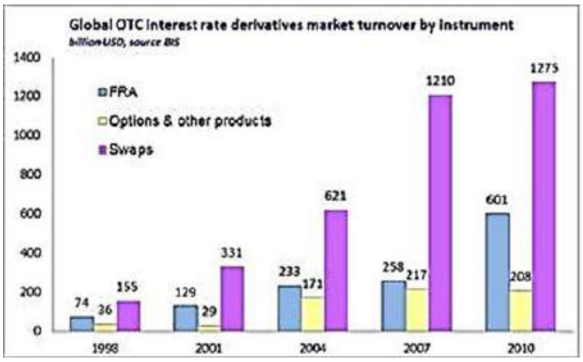
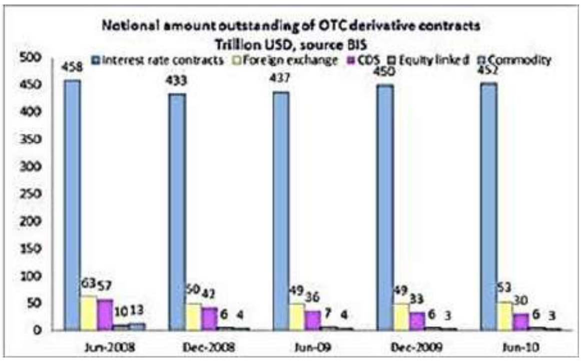
[3] In the 1st Volume, chapter 1, paragraph 2.2.8.2, we have taken the example of the two OATs issued by Agence France Tresor on the same 25/10/2019 maturity. The first 2019 OAT was initially launched on a 30 year maturity and pays a coupon of 8.5% and the second one, initially launched on a 10 year maturity, pays a coupon of 3.75%. In March 2010, due to this large coupon differential, the duration of these two OATs was respectively set at 7.19 years for the first issue and at 8.10 years for the second one. Consequently, in a very steep yield curve environment, this large difference of duration can explain the market yield spread of these two issues, the 1st issue being at that time priced at 3.23% and the 2nd one at 3.34%.

[4] In practice this yield spread between two specific bonds bearing the same issuer and the same maturity can also correspond to different market liquidity factors attached to these bonds.

[5] http://www.bis.org/publ/rpfx10.pdf

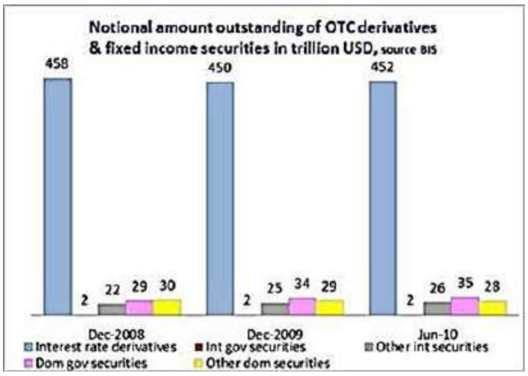
- **Notional outstanding amount:** the 2nd chart shows, that at the same time, the total outstanding notional amount of interest rate derivatives reached around USD 450 trillion.

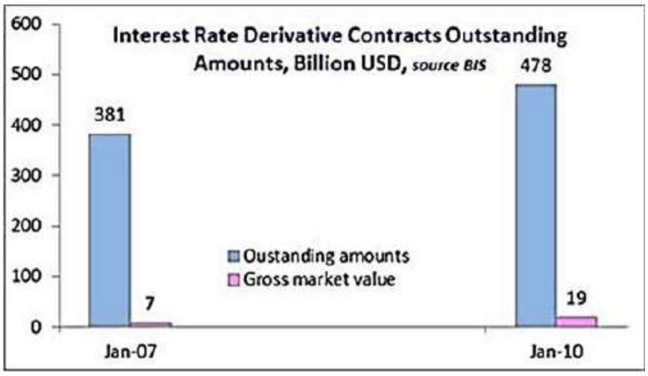




- **Comparison between worldwide IRSs and fixed income securities outstanding amounts:** The 3rd chart compares the total IRS notional outstanding amount with the total amount of worldwide public and private fixed income securities. The IRS huge outstanding amounts correspond to the OTC nature of derivative. They are based on private agreements that are not limited by the size of the underlying instruments to which they refer. In theory, in the absence of a specific regulation, their outstanding notional amount could grow indefinitely. Also hedging IRS or cancelling them is usually made by contracting new IRS, hence increasing the outstanding notional numbers.

- **Comparison between the total IRS outstanding amounts with their much smaller market value,** as shown on the 4th chart: The BIS survey explains that the gross market value of IRSs is by far much lower than their total outstanding amount and provides a better estimate of their counterparties' market exposure. The Gross Market Value expresses the mark to market of reporting dealers' interest rate swap outstanding amounts and as noticed by the BIS, the counterparty risk on these positions is reduced by bilateral netting and collateral arrangements.





- Its valuation is much more precise than the one of the actuarial yield to maturity formula and its hedging can be secured using zero coupon interest rate swaps.

- Par bonds do not offer to investors the same quality of hedging. In order to receive at the bond maturity the ex-ante actuarial yield corresponding to the acquisition price of the bond, investors would have to reinvest over the life of the bond, the fixed coupon received on each coupon payment date, at a yield to maturity rate equal to the bond actuarial yield calculated ex-ante. Zero coupon bonds solve this problem, but we have seen that these bonds are not so common.

- To the contrary, zero coupon interest swaps are very liquid, since they are at the essence of interest rate derivatives' pricing.

3. The more precise valuation of the zero coupon methodology

We will later on develop that:

- On a same fixed income instrument quoted at par, the actuarial formula and the zero coupon methodology calculate an identical yield.
- But, in the case of a same fixed income instrument bearing a price different from par, the two methodologies produce different yields.
- The zero coupon methodology allow for a better yields comparison between bonds bearing different market prices and IRSs and can propose an adequate hedge on a wide variety of cash flows' structures.

4. The different nature of IRS and fixed income securities

Although used as the common reference of pricing of fixed income securities, interest rate swaps and fixed income securities bear a very different nature:

- **IRSs and fixed income securities present a very different risk profile:** we will later on detail IRS credit risk, explain that this credit risk can be split into two different credit components:

  1) The swap counterparty risk, over the life of the interest rate swaps, for instance 10 years. However this counterparty risk is in fact strongly reduced due to the collateralization of IRSs,

  2) And the counterparty risk associated to IRS floating rate references, for instance the 6 month Euribor money market underlying credit exposure. This counterparty risk is limited to the maturity of the floating reference since, on each fixing dates, the panel contributing to Euribor fixings excludes banks that are in default.

- **The increased credit risk and liquidity premium of long term bonds:** A well rated bank launching a 10 year bond will pay higher credit and liquidity spreads than the ones it pays on its 6 month borrowings.

5. Main types of interest rate swap and derivatives described in this document

Interest rate and currency derivatives can be used to calculate and potentially arbitrage the relative value of a very large spectrum of financial market instruments, currency swaps, fixed income securities, commodities and equity portfolio for example. However, this publication does not intend to describe extensively each of the derivative instruments available on the market. It concentrates on the most frequent and plain vanilla interest rate derivatives associated with the issuance of long term fixed income securities, either from an issuer point of view or an investor point of view. We will mainly describe:

- **Plain vanilla fixed to floating IRSs bearing a money market reference:** These swaps constitute by far the largest segment of IRS derivatives. Their financial characteristics, the construction of the zero coupon curve and of the forward rates are largely described and we expose how to price plain vanilla fixed income securities against the Euribor swap curve.
- **Inflation linked swaps:** Since the introduction of the first OATi in 1998, this market has strongly developed. We describe their specific financial characteristics and their valuation methodology.

- **CMS or CMT swaps:** We will not detail the exotic structures based on these long term references, for instance the spread between the 2 year CMS and the 10 year one, even if these types of structures were very "fashionable" before the subprime crisis. We will nonetheless concentrate on the pricing methodology of plain vanilla CMS or CMT swaps.

- **Plain vanilla interest rate basis swaps and currency swaps:** These swaps are also very commonly used in the market. We will show how a corporate issuer launching a USD fixed rate bond issue can transform its exposure into a synthetic liability in EUR. Since August 2007, the valuation of these basis swaps, either denominated in two different currencies like a EUR/USD basis swaps or in the same currency like a Euribor 6 month against Euribor 3 month basis swap, has largely changed and we explain these instruments' current valuation.

- **Price relationship between IRSs, fixed income securities and plain vanilla interest rate options:** Fixed income securities embedding a plain vanilla or structured interest rate option have also been very common over the last years. We will not describe in detail these synthetic securities; however, we explain the valuation basic pricing principles of interest rate caps, floor and plain vanilla swaptions.

Chapter 1 – Plain vanilla Interest Rate Swaps

1. General characteristics of IRS markets

1.1 Interest rate swaps' definition

An interest rate swap is a contract where two counterparties agree to exchange, swap, the streams of future interest rate payments based on different market conventions, for a specific notional principal amount and a given maturity. This Over the Counter Contract, OTC, can be tailor made so as to match its counterparties' requirements.

In its most basic form, a standard, also called "plain vanilla" interest rate swap, exchanges fixed-rate interest payments for floating-rate interest payments. Each of these fixed and floating streams of payments of the swap are referred to as its two legs, the fixed leg one and the floating leg one.

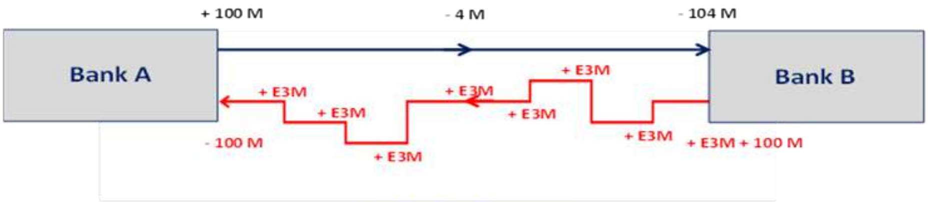
1.2 Interest rate swaps and bonds' relationship

- From a pure financial point of view, a plain vanilla swap can be analyzed as the exchange of two bonds bearing a same nominal amount and a same maturity, one counterparty issuing a fixed rate bond ant the other one a floating rate one. These instruments are similar to the fixed income securities described in the 1st volume of this document.

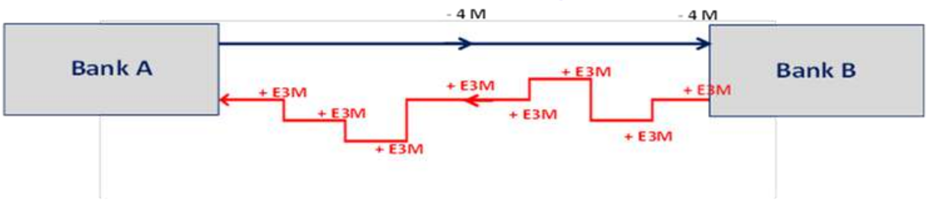
- In theory, this exchange of bonds between two counterparties could be qualified as parallel (or back to back) loans and, in the case of the failure of one the two counterparties, the non-failed counterparty has to repay the principal amount it had borrowed at maturity, even if the failed counterparty does not repay the principal amount of its debt.

- From a pure legal point of view, in order to reduce IRS counterparty risks, the swap principal amount (used to calculate the interest payments paid and received by each counterparty) is defined as a notional amount that is never exchanged between these two counterparties: In an IRS transaction, these opposite principal amounts at inception of the swap transaction and at its maturity are netted[6]. The two following charts compare the different cash flows of a back to back loan and of an interest rate swap paying a 4% fixed rate against 3 m Euribor, on a 2 year maturity.

Back to back loan



Interest rate swap



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[6] Under a contractual netting agreement, two counterparties agree to exchange the net difference between payable and receivable cash flows falling on a same date, so as to reduce their counterparty risk.

- However, from a financial point of view, the price of an IRS valorizes separately the cash flows of its fixed and floating legs and considers that the notional principal amount that has been netted in the swap contract, has been in fact exchanged between the two counterparties on each leg, at the initiation of the swap and at its maturity.

1.3 Interest rate swaps' language

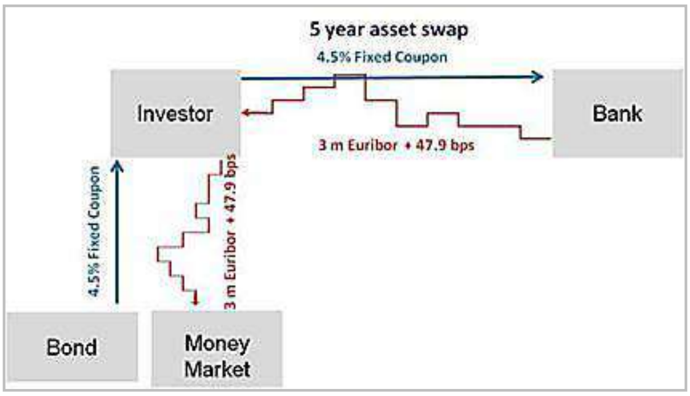
So as to ensure that the swap transaction will not be legally re-qualified as a back to back loan, interest rate swap markets have adopted a specific "esoteric" language based on the sole streams of their interest payments. Interest rate swap main market risk lies on its fixed leg, due to its higher duration compared to that of its floating leg, and by convention in a vanilla swap, each counterparty risk is expressed in relation with the swap fixed leg:

- The counterparty defined as the "payer" pays the streams of the swap fixed leg and receives the streams of the swap floating leg,

- The counterparty defined as the "receiver" receives the streams of the swap fixed leg and pays the streams of the swap floating leg.

1.4 Interest rate swaps' main utilization in connection with fixed income securities

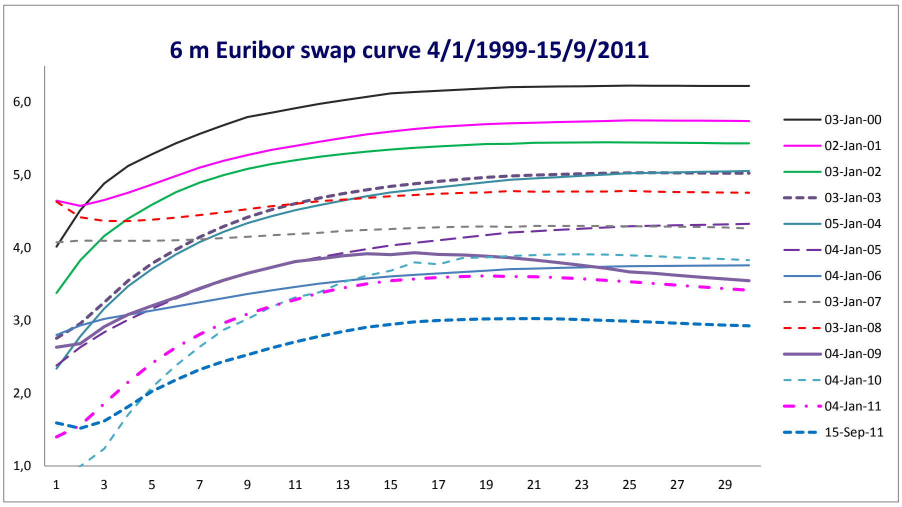
Interest rate swaps are used to transform the original cash flows of an asset or liability, by converting for instance the fixed rate cash flows received on a bond into floating rate cash flows generally based on a money market reference. The following example briefly illustrates how a bond issuer that has paid a 4.5% fixed rate over 5 years to its investors can convert its fixed rate position into a floating one at Euribor + 48 bps[7].



[7] In this example we assume that the two year swap against Euribor flat is priced at 4% and we translate the 50 bps premium paid by the issuer on its 4.5% bond into Euribor + 50 bps, by simply comparing the two fixed rates. In the case of bonds priced at par, this approximation can be considered as a fairly good one:
- The fixed rate bond pays its coupons annually on an actual/ actual basis and we suppose that the fixed rate leg of the interest rate swap hedging this position adopts the same convention. However, the 3 m Euribor floating leg of this swap is paid quarterly on an Actual/360 basis. In theory in this example, using a pure actuarial calculation, the annual 4.5% fixed rate yield of the bond should be equivalent to Euribor + 48 bps paid quarterly on an Actual/360 basis.
- In practice, the bond issuer will have to pay a transacting cost to its swap counterparty.

1.5 Illustration of the changes in interest rate swaps over the last 10 years

The chart following recalls how 1-30 year 6 months Euribor € interest rate swaps have behaved since January 20000. Over this period, the spread between the highest curve in January 2000 and the lowest one in September 2011 has exceeded 3.2% on the 5, 10 and 30 years' maturity.



Given this high level of interest rates' volatility and the potential long maturity of IRS transactions, a naked interest rate swap transaction can create a huge exposure to a change in market conditions.

The table following illustrates the potential mark to market gain or loss on the fixed leg of an IRS, with a +1% change in rates, on maturities ranging from 5 years to 30 years. This exposure is not fundamentally different from the one that a trader can create on the bond market, but a derivative transaction does not involve the payment of its principal amount.

Jan 2001 - Sep 2011, Change in price in % of notional amount due to a 1% change of rate on 5 to 30 year maturity

Maturity	5	10	15	20	30
Highest curve 03/01/2000	5,29	5,855	6,123	6,208	6,223
Lowest curve 15/09/2011	2,03	2,623	2,946	3,024	2,297
Mark to Market + 1% higher curve	**-4,3%**	**-7,4%**	**-9,6%**	**-11,3%**	**-13,4%**
Mark to Market -1% lower curve	**4,7%**	**8,7%**	**12,0%**	**14,8%**	**21,5%**

As shown on the previous page chart, since January 2000, the € swap curve have been most of the time positively sloped and some economists conclude that a steep yield curve predicts an increase in future interest rates. The forward rates derived from a steep spot yield curve are effectively priced at a higher yield than this spot curve, as shown on the chart following

However, the previous page chart illustrates that over the last 10 years, contrary to these potential predictions, the € 1-30 swap curve has overall sharply decreased. Consequently since market conditions tend to change, one should be cautious over the concept of the "predictive effect of the forward curves".

2. IRS market conventions and basic valuation principles

We recall that the valuation of an interest rate swap supposes the reintroduction of its netted swap notional amount into each swap legs, as if we were pricing a back to back loan, cf. the chart shown on paragraph 1.2.

2.1 Interest rate swaps' market conventions mainly depend on their floating reference

In euro, fixed to floating plain vanilla interest rate swaps are usually referenced to two different types of money market references, the Euribor reference and the Eonia one, both references being also used in the floating rate bond markets.

- IRSs' floating leg conventions are identical to the ones applicable to the bond floating rate conventions that we have described in the 1st Volume of this document.
- The market conventions applicable to IRSs' fixed leg are not strictly identical to the market conventions applicable to the bond market. And these fixed leg market conventions differ, depending whether their floating reference is indexed to Euribor or Eonia.
- In addition, like with fixed income securities, long term IRSs and short term ones also bear different market conventions.

2.1.1 Long term standard market conventions applicable to IRS fixed legs

The two following paragraphs describe the standard market conventions usually applicable to the fixed legs of plain vanilla swaps.

However, since interest rate swaps are traded in the OTC market, the conventions applicable to a customer driven swap are often adapted to the needs of a specific customer. For instance the fixed leg of an IRS contracted in connection with a bond issue will generally adopt the Actual/Actual day count basis associated with bond issues, where the standard interbank interest rate swap fixed leg use the 30/360 day count basis.

2.1.1.1 Market conventions applicable to long term Euribor IRS fixed legs

- Standard Euribor swaps settle on T+2 according to Target business days.

- Accrued interest day count basis: Following ISDA recommendations, long term fixed rate accrued interest is calculated according to the 30 E/360 basis[8].

- Business day conventions: fixed rate coupons falling on a non-business day are paid on the next following business day and their amount of interest is unadjusted.

- In euro, the Euribor fixed leg rate of a standard IRS is by convention expressed as a yearly rate and its coupons are paid once a year.

- In the case of a first irregular coupon period, the yearly rate applicable to this 1st coupon is equal to the rate of the subsequent coupons without decompounding it, and it is therefore expressed as a simple rate.

2.1.1.2 Market conventions applicable to long term Eonia IRS fixed legs

- Like with Euribor swaps, the euro fixed leg rate of a long term standard IRS is by convention expressed as a yearly rate paid once a year and in the case of a first irregular coupon period its 1st coupon rate is like with Euribor swaps equally applied as a simple rate.

- Unlike Euribor swaps, standard Eonia swaps settle on T+0 according to Target business days.

- Standard long term Eonia fixed leg rate accrued interest is calculated according to the Actual/360 basis.

- Business day conventions: fixed rate coupons falling on a non-business day are paid the next following business day and, on Eonia swaps, the amount of interest is adjusted.

2.1.2 Market conventions applicable to IRS Euribor floating legs

Market conventions applicable to the Euribor leg of an IRS are strictly identical to the ones of money market deposits described in the first volume of this document under chapter 1 paragraph 4.1.

- Standard Euribor swaps settle on T+2 according to Target business days.

- Like with money market instruments, Euribor fixings are quoted as simple yield expressed as a yearly rate.

- Interest is paid at maturity of the coupon period and is calculated according to the Actual/360 day count fraction applicable to euro money markets.

- Business day convention; the IRS Euribor floating leg uses the modified following convention described in the first volume chapter 1 and, if the number of days in the interest period is modified, interest payment is adjusted.

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[8] In spite of the choice of an Actual/actual day count basis on the bond market in euro, ISDA indicated in 1998 that the 30E/360 ISDA convention was used as the standard convention for DEM and XEU swaps before the introduction of the euro. It also indicated that swap and bond conventions do not need to be identical. Market conventions in the United States, for example, are not identical on the swap market that adopts the Actual/360 basis and the Treasury market using the Actual/actual basis.

- 6 month Euribor: Euro IRS bearing an initial maturity of 2 years and more are generally referenced to 6 month Euribor fixings. This floating rate convention was promoted before the introduction of the euro by ISDA, the International Swap and Derivative Association, and some other financial associations, with the objective of reducing the counterparty risk of interest rate swaps. These associations were arguing that euro government bonds (and euro interest rate swaps) should switch from yearly coupon payments to semi-annual coupon payments, a periodicity already adopted by US and UK government bonds.
  Matching the periodicity of payments of both the fixed and the floating rate of interest rate swap should have allowed for the netting of these coupon streams and therefore reduced the counterparty risk. But euro sovereign issuers that were in majority paying yearly coupons did not adopt this semiannual convention[9].

- 3 month Euribor: However 3 month Euribor fixings that are generally used on short term Euribor swaps can also be used on long dated swaps.

2.1.3 Market conventions applicable to IRSs' Eonia floating legs

- Standard Eonia swaps settle on T+0 according to Target business days.

- Business day conventions: Eonia floating rate coupons falling on a non-business day are paid the following business day and in this case, the amount of interest is adjusted.

- The Eonia floating reference is compounded every working day according to the OIS convention defined by Euribor ACI[10] and it is usually paid on a yearly basis, according to the following formula:

EURIBOR

EURIBOR ACI

For calculating the variable rate the following formula shall be applied.

$$r = \frac{360}{n}\left[\prod_{i=ts}^{te-1}\left(1+\frac{ri * di}{360}\right)-1\right]$$

r	Variable rate taking compound interest into account
t_s	Start date of the EONIA swap
t_e	End Date of the EONIA Swap
ri	EONIA fixing rate on the i-th day
di	Number of days that the value ri is applied (normally one day, three days for weekends)
n	Total number of days

[9] Twelve years after the introduction of the euro, the choice of this 6 month Euribor fixing is often contested:
- The USD interest rate swap market is referenced to 3 month Libor,
- USD and EUR money market futures bear a 3 month maturity,
- The liquidity of unsecured 6 month deposits in euro, supposed to be the underlying instrument of 6 month Euribor fixings is largely questionable.

[10] http://www.aciforex.org/docs/markettopics/EONIA_SWAP_INDEX_BrochureV2-2008-01044-01-E.pdf

- The OIS compounded rate described above is applied to the notional amount of the IRS transaction for the calculation of the interest payment on the Eonia floating leg. It should be noted that the above calculation already includes the Actual/360 money market basis and the variable rate *r* is simply multiplied by the notional amount of the swap transaction to obtain the value of the floating rate payment.

2.1.4 Short term IRSs market conventions

Short term interest rate swaps, IRSs bearing a maximum maturity of 1 year, use the same market conventions and interest payment calculation as the ones of money market instruments defined in the first volume of this document:

- Their fixed rate leg interest payments are calculated by using a simple yield expressed as a yearly rate and an Actual/360 day count basis.

- And their floating leg interest payments are referenced either to the 3 or 6 month Euribor, the 3 month being mostly used, or to the Eonia reference using the same market conventions as the ones applicable to these money market references.

2.1.5 Formulae applicable to IRSs' fixed and floating coupons

The calculation of fixed or floating rate coupons applicable to interest rate swaps is much simpler than the one of bonds since with interest rate swaps, the notion of denomination does not exist. Coupons formulae in percentage and in € are directly applied to the swap notional amount, for the calculation of full coupons or accrued interest.

This calculation depends on the swap market conventions applicable on the fixed and floating leg of the swap, coupon periodicity and day count basis.

2.1.6 The yield relationship between par interest rate fixed and floating legs

As already explained, the respective value of the fixed leg of a par swap and that of its floating leg remains equal, whatever the changes in market conditions; The yield relationship between these two legs can be seen as a chicken and egg problem. For instance, in the case of a sudden rise in anticipated inflation:

- This change of market previsions can directly result in a rise of long term rates,
- Or in a decrease in long term interest rates, if the market prices an immediate strong increase of short money market rates by the central bank, that will oppose to the rise of anticipated inflation.
- However, independently of the effects of these anticipations on the shape of the swap curve, any change in market anticipations is directly transmitted to the yield of the fixed and floating legs of a par swap.

In order to describe the different factors influencing par interest rate yields, we analyze separately the valuation of each of the legs of a par swap; Its fixed leg valuation is described under paragraph 3 and, its floating leg valuation under paragraph 4.

3 Par interest swaps fixed rate leg valuation

The paper briefly indicate the pricing methodology of fixed income securities based on the par interest swap curves published on the screens of the main interbroker dealers. The complex methodology of constructing theses par curves using different fixed income instruments is not detailed extensively, since it has been largely documented over the years. However we give a short description of the basic principles applicable to the construction of this par curve, so as to better understand the valuation of fixed income securities against an interest swap curve.

IRS swap curves' construction uses a "bootstrap" methodology based on the relationship between the yields quoted in the markets on the most liquid benchmark interest rate instruments available on different maturities: Money market deposits and futures, interest rate swaps and government bond futures.

However the maturity of these different instruments and the credit of their issuers being not homogeneous, these various instruments' yields have to be linked by using various interpolation algorithms.

This resulting calculated yield curve is then smoothed so as to derive from this swap par curve very regular zero coupon and forward swap curves.

This curve also incorporates interest rate swaptions' volatility, so as to avoid any convexity arbitrages. At this stage we simply notice that, due to the inclusion of swaptions' volatility, interest rate swap curves can present an inversion on long maturities and this "technical" inversion should not necessarily be interpreted as the early signs of an economic recession, as very often indicated by some economists[11].

[11] The Federal Bank of New York publishes a model that uses the difference between 10-year and 3-month Treasury rates to calculate the probability of a recession in the United States twelve months ahead: http://www.newyorkfed.org/research/capital_markets/Prob_Rec.pdf
"The (Treasury) yield curve… is typically upward sloping and somewhat convex. At times, however, it becomes flat or slopes downward ("inverts,"in Wall Street parlance), configurations that many business economists, financial analysts, and other practitioners regard as harbingers of recession" Predicting Real Growth Using the Yield Curve by Joseph G. Haubrich and Ann M. Dombrosky .

3.1 Spot par swap curves

3.1.1 Par swap curve definition

A par swap curve refers to a fixed rate swap curve that is only composed of individual par swaps, swaps bearing, like a par bond, a fixed coupon equal to their yield to maturity.

3.1.2 Interbroker live par swap rate quotations

Primary swap market rates, quoted real time by interdealer brokers, correspond to the fixed leg rate of a par swap: Each yearly step of the interest rate par swap curve is generally quoted on maturities ranging from 1 year to 30 year by interdealer brokers like ICAP, Tullet, Tradition and others. These rates are published by the main financial information providers and financial newspapers. For instance, historical data based on ICAP[12] interest rates swap curves at close of business are available on the Financial Times web site : http://markets.ft.com/research/Markets/Bonds .

The Reuters page following illustrates the fixed leg rates of the € par swap curve against 6 month Euribor quoted by ICAP in January 2009. On top of the 1 year to 30 year rates, this page also publishes the rates on 35, 40, 50 and 60 years.

These rates are currently quoted with 3 decimals.

14:40 28JAN09 ICAP
Euribor vs 6 mth
Sp

1 Yr	2.159-2.109	16Yrs	4.029-3.979
2 Yrs	2.334-2.284	17Yrs	4.045-3.995
3 Yrs	2.619-2.569	18Yrs	4.052-4.002
4 Yrs	2.860-2.810	19Yrs	4.050-4.000
5 Yrs	3.045-2.995	20Yrs	4.039-3.989
6 Yrs	3.209-3.159		
7 Yrs	3.349-3.299	21Yrs	4.022-3.972
8 Yrs	3.470-3.420	22Yrs	3.999-3.949
9 Yrs	3.579-3.529	23Yrs	3.973-3.923
10Yrs	3.679-3.629	24Yrs	3.947-3.897
		25Yrs	3.921-3.871
11Yrs	3.766-3.716		
12Yrs	3.843-3.793	26Yrs	3.898-3.848
13Yrs	3.909-3.859	27Yrs	3.877-3.827
14Yrs	3.960-3.910	28Yrs	3.858-3.808
15Yrs	4.000-3.950	29Yrs	3.841-3.791
		30Yrs	3.823-3.773
		35Yrs	3.735-3.685
		40Yrs	3.658-3.608
		50Yrs	3.596-3.546
		60Yrs	3.535-3.485

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Figures in red indicate the decimal changes in the yield of the yearly par swap rates that are currently updated. As we will later explain, since this curve has to be as smooth as possible, in order to produce a very regular forward curve, a change at any yearly point of the par curve is immediately reflected into the other yearly rates of the curve.

[12] ICAP is one of the leading interdealer brokers in the interest rate derivative markets. The French Bond Association publishes monthly a zero coupon curve for corporate accountancy purposes that is based on the Euro swap par curve published by ICAP.

3.1.2.1 By construction, the yield of the fixed leg of a par swap is equal to its floating leg yield

When analyzing the floating cash flows of a primary par swap, a not yet transacted swap, we will see that their anticipated compounded yield is equal to its the fixed leg yield; The yield of these "anticipated" forward rates is derived from the yield of the fixed leg and none of the primary par swap floating references have been yet determined.

For this reason, the rate of a primary par swap is solely expressed as a fixed rate attached to a specific floating reference, 3 or 6 month Euribor or Eonia, for instance. The value of its 1st floating reference will be fixed at the inception of the swap[13].

The table opposite illustrates the theoretical cash flows of each leg of a par 100 M 10 year 3.7698% fixed rate / 6 month Euribor not yet contracted. In this example, the swap par curve is published with 4 decimals. These cash flows include the principal amount of the fixed leg and the floating leg at the swap settlement date and at its the maturity date, even if on a swap transaction the cash flows corresponding to the notional amount are netted and therefore not exchanged.

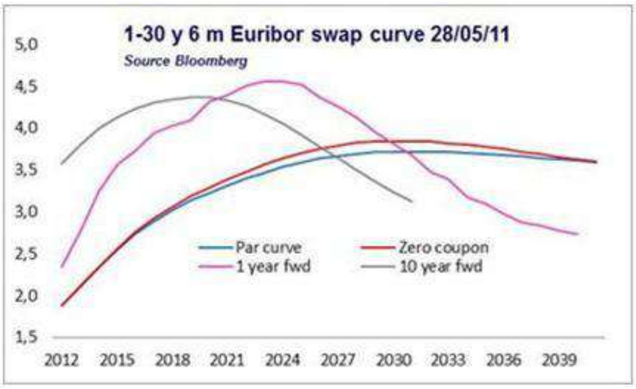
Cash flow of a 10 year / 6 m Euribor par swap not yet contracted

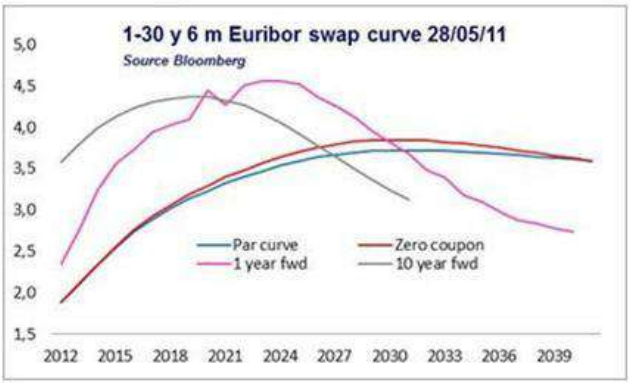
Settlement	01/06/2009	Maturity	01/06/2019	Fixed rate 3,7698%
Not Amount	100 M		Interest payments	
Years	Par curve	Payment dates	Fixed leg	Floating leg
0		01/06/2009	-100 000 000	100 000 000
0,5	1,4640	01/12/2009		- 6 m Euribor
1	1,4700	01/06/2010	3 769 800	- 6 m Euribor
1,5	1,6190	01/12/2010	0	- 6 m Euribor
2	1,7680	01/06/2011	3 769 800	- 6 m Euribor
2,5	1,9885	01/12/2011	0	- 6 m Euribor
3	2,2090	01/06/2012	3 769 800	- 6 m Euribor
3,5	2,4075	01/12/2012	0	- 6 m Euribor
4	2,6060	01/06/2013	3 769 800	- 6 m Euribor
4,5	2,7624	01/12/2013	0	- 6 m Euribor
5	2,9188	01/06/2014	3 769 800	- 6 m Euribor
5,5	3,0447	01/12/2014	0	- 6 m Euribor
6	3,1705	01/06/2015	3 769 800	- 6 m Euribor
6,5	3,2694	01/12/2015	0	- 6 m Euribor
7	3,3683	01/06/2016	3 769 800	- 6 m Euribor
7,5	3,4482	01/12/2016	0	- 6 m Euribor
8	3,5280	01/06/2017	3 769 800	- 6 m Euribor
8,5	3,5938	01/12/2017	0	- 6 m Euribor
9	3,6595	01/06/2018	3 769 800	- 6 m Euribor
9,5	3,7147	01/12/2018	0	- 6 m Euribor
10	3,7698	01/06/2019	103 769 800	- 6 m Euribor - 100 M

The calculation methodology of the zero coupon rates and the forward rates are derived from the sole value of the fixed rate leg of a par swap. This methodology is homogeneous with the one that we have described in the 1st volume for the STRIPS of fixed government bonds. But government bond curves are based on the yield of bonds that are not priced at par and the zero coupon and forward rates that they produce are far from being smooth.

3.1.2.2 The fixed leg of a par swap curve produces very smooth zero coupon and forward rates

The 1st chart below shows the zero coupon, 1 year forward and 10 year forward curves that are derived from the 28/05/2011 par swap curve.





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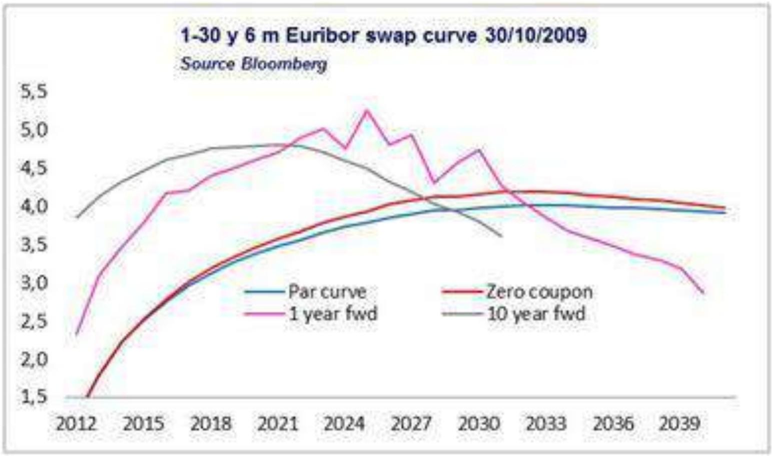
[13] Like with Euribor swaps, Euribor fixings settle at T+2 and, under standard market conventions, the 1st Euribor fixing is usually fixed on a new transacted swap according to the fixing of the Euribor reference determined at 11:00 a.m. CET on its transaction date. Before, 11 a.m., the fixing is not yet determined, after 11 a.m. the counterparties usually use the same day 11 a.m. fixing already determined.

The second one illustrates the change in the 1 year forward rate consecutive to a 1 bp. shock applied to the 10 year maturity of the par curve.

In practice, with a 5 bps bid offered spread on the par curve, forward rates are nowadays not so regular. Since the subprime crisis, 1-30 year par swap curves that were previously quoted with a 3 bps bid offer spread are now quoted with a 5 bps spread. Depending on supply and demand on the par curve, the 1 year forward rates that we use as an indicator of the smoothness of the par curve, can result in a quite erratic 1 year forward curve.

The granularity of 1 year forward rates is amplified by its short underlying forward maturity, while 10 years forward rates show a more regular shape, as indicated on the chart below.



3.1.3 Par curve interpolation

3.1.3.1 Interpolating missing yearly points of the par curve

The € ICAP swap curve publishes the yearly par yield curve, from 1 year to 30 years. On longer maturities, it quotes the rate of 35, 40, 50 and 60 years. However, very often, some other information providers in € or in other currencies, only publish above 10 years, the rates of 12, 15, 20 and 30 years. In order to build regular zero coupon and forward curves, the yearly missing points of the par curve have to be interpolated.

Roughly speaking, two main methods can be used for the interpolation of a yield curve:

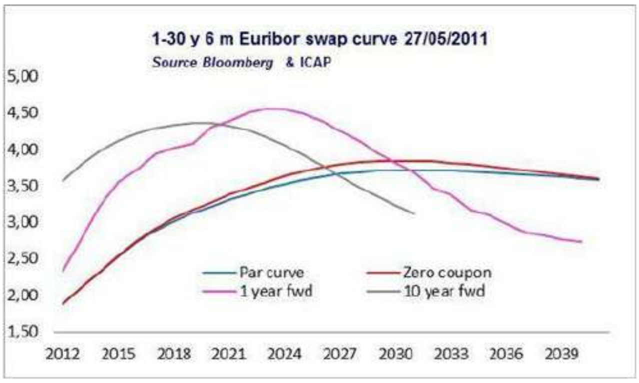
- The basic one consists to draw a straight line between two known points of the curve, in order to calculate the curve missing points. However, in the case of a very steep curve and of yearly par swap missing points, this method does not produce sufficiently smooth zero coupon and forward rates,
- Different other methods of interpolation can be used to calculate the par swap curve yearly missing rates, so as to reproduce its curvature and to contribute to its smoothness. Some of these different methods are published on the following web sites[14].

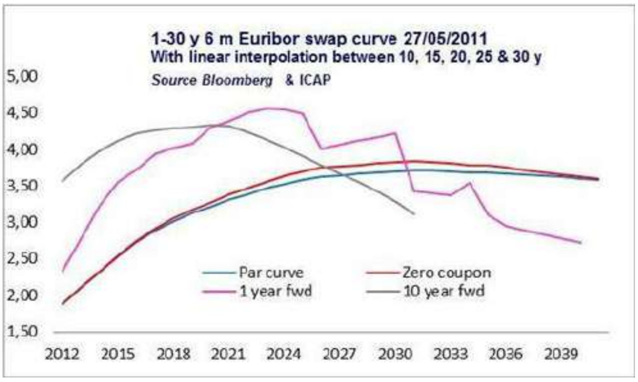
[14] Bank of Canada: http://www.bankofcanada.ca/wp-content/uploads/2010/01/wp00-17.pdf
BIS : http://www.bis.org/publ/bppdf/bispap25.pdf

- The CNO has defined a simple and efficient cubic interpolation yield curve methodology[15]. We use this cubic interpolation method in this document.

The 1st chart following illustrates the 1 and 10 year forward swap rates calculated from the yearly steps of the 1-30 year swap curve published on 28/05/2011.

The 2nd chart following shows the resulting 1 and 10 year forward swap rates calculated from the same par swap curve, by ignoring some intermediate par rates over 10 year and replacing them with a linear interpolation between 10, 15, 20, 25 and 30 year rates.





3.1.3.2 Interpolating missing rates on broken interest rate periods

Every working day, € interdealer brokers' screens quote the yearly steps of the plain vanilla fixed interest rate par swap curve from 1 year to 30 years.

IRSs bearing a life to maturity that does not correspond to an entire number of years[16] are treated as non-standard IRSs bearing a broken period and a broken maturity date.

A swap transacted on a given settlement date with a maturity equal to a whole number of years will be treated as a non-standard swap bearing a broken maturity date during the next 364 days and it will have to be mark to market during this period by using an interpolation of the two yearly rates that surround its maturity. Consequently we have to choose how to interpolate the yield between two yearly steps of the curve, making either a linear interpolation or a more sophisticated one, replicating the curvature of the yield curve.

The choice of the interpolation methodology between two yearly steps of the swap curve is of course less important than the one used to interpolate rates bearing a 5 year difference of maturity. However with a steep yield curve, the results between a linear interpolation and a cubic one can give quite different results, when calculating the price of a swap bearing a broken maturity. Chapter 2, paragraph 1.2.1.2, we propose an interpolation methodology between the yearly steps of the par curve.

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[15] The CNO cubic interpolation method, an interpolation by polynomials of degree 3, provides continuous first derivatives at points known:
http://www.cnofrance.org/IMG/pdf/Methodologie_de_calcul_des_taux_zero_coupon_CNO.pdf

[16] Or the standard interest rate coupon periods, in the case of swaps paying a fixed coupon on a periodicity inferior to 1 year.

3.2 Calculation of the zero coupon swap curve basic principles

The mathematical methodology for calculation a zero coupon swap curve is described in CNO-FBA Indices' chapter under the title: "Method for calculating a CNO Zero Coupon Yield Curve".

3.3 calculation of long term forward par swap curves

3.3.1 Par forward swap curve definition and conventions

Definition: A forward swap is an interest swap starting on a specific agreed date in the future, for instance a 5 year swap starting in 1 year. A forward par swap curve indicates that the forward swap rates derived from a spot par curve bears a forward coupon that is equal to their forward yield.

Market conventions: Forward swaps use the same market conventions as their spot equivalents. In the forward swap language, a 10X12 forward swap quotes the rate of a 2 year swap starting in 10 years, and a 10X30 forward swap quotes the rate of a 20 year swap starting in 10 years.

3.3.2 IRS forward swap transactions

IRS forward swaps are largely traded in the interbank swap market and their par forward rates are published on the screens of the main interdealer brokers, as seen on the Reuters page following.

A forward swap is financially equivalent to the combination of two different spot swaps on different maturities (one being a payer swap and the other a receiver swap) and it can be replicated by entering into these two different swap transactions. However, in the major currencies, a liquid forward swap market has developed minimizing the transaction costs.

In the interbank swap market, forward swaps are largely used to manage the duration of swap portfolios at an economic cost.

- The ICAP Reuters page opposite publishes the 10X12 to 10X60 forward swap rates spreads that are illustrated on its red box.

14:40 28JAN09 ICAP UK69580 ICAPEURO
Euribor vs 6 mth 3/6 basis
Spot Starting Date

1 Yr	2.159-2.109	16Yrs	4.029-3.979	1 Yr	+17.8		
2 Yrs	2.334-2.284	17Yrs	4.045-3.995	2 Yrs	+11.5		
3 Yrs	2.619-2.569	18Yrs	4.052-4.002	3 Yrs	+8.60		
4 Yrs	2.860-2.810	19Yrs	4.050-4.000	4 Yrs	+7.00		
5 Yrs	3.045-2.995	20Yrs	4.039-3.989	5 Yrs	+6.00		
6 Yrs	3.209-3.159			6 Yrs	+5.20		
7 Yrs	3.349-3.299	21Yrs	4.022-3.972	7 Yrs	+4.60		
8 Yrs	3.470-3.420	22Yrs	3.999-3.949	8 Yrs	+4.20		
9 Yrs	3.579-3.529	23Yrs	3.973-3.923	9 Yrs	+3.80		
10Yrs	3.679-3.629	24Yrs	3.947-3.897	10Yrs	+3.50		
		25Yrs	3.921-3.871				
11Yrs	3.766-3.716						
12Yrs	3.843-3.793	26Yrs	3.898-3.848			10X12	0.184/0.144
13Yrs	3.909-3.859	27Yrs	3.877-3.827			10X15	0.341/0.301
14Yrs	3.960-3.910	28Yrs	3.858-3.808			10X20	0.380/0.340
15Yrs	4.000-3.950	29Yrs	3.841-3.791			10X25	0.262/0.222
		30Yrs	3.823-3.773			10X30	0.164/0.124
		35Yrs	3.735-3.685			10X35	0.076/0.036
		40Yrs	3.658-3.608			10X40	-0.001/-0.041
		50Yrs	3.596-3.546			10X50	-0.063/-0.103
		60Yrs	3.535-3.485			10X60	-0.124/-0.164

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- Instead of quoting forward swap rates in absolute rates, these screens quote forward swap rates at a positive or negative spread that has to be added to the par swap rate corresponding to the start of the forward period: For instance on this page, the 10X60 forward bid offer spread of - 0.124/ - 0.164 indicates the spread that has to be added to the 10 year spot rate to obtain the rate of 50 year forward swap starting in 10 years. In this example this spread has to be deducted from the 10 year bid offer spread, since it is negative, In the B to C market, forward swaps are used to fit the specific needs of clients wanting to hedge their interest rate exposure on long term transactions. For instance:

- An airline company can fix the borrowing rate used to finance a new plane, at the signature of its acquisition, well before the delivery of the plane.

- In the primary bond market, the settlement date of a new issue is longer, for instance two weeks, than the standard T+2 settlement date of Euribor swaps. With a forward swap, for instance the 5 year in two weeks, an issuer launching a new 5 year fixed rate bond can hedge its interest rate swap position by entering, at launch of the bond issue, into a Fixed to Floating IRS even if the bond transaction settles two weeks later.

3.3.3 Calculating forward par swap rates

3.3.3.1 The spot and forward curves' relationship

The Excel file opposite compares the valuation of a 3.77% 10 year par bond using the actuarial and the zero coupon, ZC, methodologies. This bond mimics the fixed leg of a 10 year par swap paying 3.77%; We price these Discount Factors by using either:

- A flat yield of 3.77%, column F,

- Or the yearly ZC rates derived from the 1 to 10 year par swaps rates, column H,

In both cases, the sum of these Discount Factors, cell E7, and, cell G7, is equal to 1, and, cells F6 and H6, their respective yield is equal to zero:

	A	B	C	D	E	F	G	H
1	6 m Euribor par and zero coupon swap curve							
2–4	Year	Par curve	Zero coupon rate	10 y bond cash flows	Actuarial Discount factor	Bond cash flows actuarial npv	Zero coupon Discount Factor	Bond cash flows zero cp npv
5				IRR		IRR		IRR
6				3,770%	DF Act	0,00%	DF ZC	0,0000%
7	0			-100,000	1,0000	-100,00	1,0000	-100,00
8	1	1,470	1,470	3,770	0,9637	3,633	0,9855	3,715
9	2	1,768	1,771	3,770	0,9287	3,501	0,9655	3,640
10	3	2,209	2,221	3,770	0,8949	3,374	0,9362	3,529
11	4	2,606	2,633	3,770	0,8624	3,251	0,9013	3,398
12	5	2,919	2,962	3,770	0,8311	3,133	0,8642	3,258
13	6	3,171	3,231	3,770	0,8009	3,019	0,8263	3,115
14	7	3,368	3,446	3,770	0,7718	2,910	0,7889	2,974
15	8	3,528	3,621	3,770	0,7438	2,804	0,7523	2,836
16	9	3,660	3,768	3,770	0,7167	2,702	0,7169	2,702
17	10	3,770	3,892	103,770	0,6907	71,674	0,6826	70,833

- The Discount Factors express the bond cash flows' present value. The difference between its 1st settlement date Discount Factor and its last Discount Factor at maturity corresponds to the present value of its yearly interest payments, independently of the pricing methodology.

- Knowing the Discount Factors' values applicable to the cash flows of a par rate bond, discounted at either a flat actuarial rate or by using the ZC methodology, we can calculate its par yield.

We have seen that the yield of a spot zero coupon is equal to:

- The difference between the present value of its 1st Discount Factor (that is equal to 1) and the present value of its maturity date Discount Factor

- This difference expresses the present value of its yearly compounded interest rate and in order to calculate a zero coupon yield, this price difference is decompounded over the zero coupon life, according to the following formula:

$$ZCy_{an} = \left(\frac{1}{DF_n}\right)^{1/n} - 1 \qquad (B.1.3.3)$$

3.3.3.2 Par forward swaps' formula

In the case of a par swap or a par swap bond, paying annual coupons, knowing the values of its yearly Discount Factors, we calculate its rate according to the following formula[17]:

$$PFS_{ny\ X\ ny'} = \left(\frac{DF_{ny} - DF_{ny'}}{\sum_{DFny'}^{DFn+1y}} \right) \qquad (B.1.3.6)$$

With :

$PFS_{ny\ x\ ny'}$	: Par Forward Swap starting at year n and maturing at year n'
DF_{ny}	: Discount Factor at Start of the Forward Swap
$DF_{ny'}$	: Discount Factor at maturity of the Forward Swap

The table following describes the calculation of zero coupon and forward par swap rates derived from the spot swap par curve:

6 m Euribor par and zero coupon swap curve, zero coupon & Forward curve

Year	01/06/2009	Par curve	Discount Factor	Zero coupon	1 year fwd	5 year Fwd	10 year fwd
1	1-juin-10	1,8875	0,9815	1,888	2,34	3,106	3,576
2	1-juin-11	2,1130	0,9590	2,115	2,78	3,428	3,802
3	1-juin-12	2,3285	0,9331	2,335	3,23	3,684	3,990
4	1-juin-13	2,5450	0,9039	2,559	3,57	3,859	4,129
5	1-juin-14	2,7370	0,8727	2,760	3,73	4,009	4,229
6	1-juin-15	2,8885	0,8414	2,920	3,94	4,143	4,305
7	1-juin-16	3,0233	0,8095	3,065	4,03	4,256	4,347
8	1-juin-17	3,1335	0,7782	3,185	4,09	4,365	4,370
9	1-juin-18	3,2250	0,7476	3,285	4,32	4,461	4,371
10	1-juin-19	3,3168	0,7166	3,388	4,39	4,501	4,331
11	1-juin-20	3,3968	0,6865	3,479	4,51	4,505	4,270
12	1-juin-21	3,4705	0,6569	3,564	4,56	4,460	4,174
13	1-juin-22	3,5358	0,6282	3,641	4,55	4,376	4,059
14	1-juin-23	3,5908	0,6009	3,706	4,50	4,258	3,923
15	1-juin-24	3,6355	0,5750	3,758	4,39	4,122	3,783
16	1-juin-25	3,6695	0,5508	3,798	4,26	3,980	3,639
17	1-juin-26	3,6940	0,5283	3,825	4,13	3,824	3,497
18	1-juin-27	3,7105	0,5073	3,842	3,96	3,675	3,364
19	1-juin-28	3,7193	0,4880	3,848	3,82	3,519	3,241
20	1-juin-29	3,7225	0,4701	3,846	3,68	3,376	3,128
21	1-juin-30	3,7213	0,4534	3,839	3,48	3,233	
22	1-juin-31	3,7143	0,4381	3,822	3,38	3,111	
23	1-juin-32	3,7053	0,4238	3,803	3,18	2,999	
24	1-juin-33	3,6918	0,4107	3,777	3,10	2,917	
25	1-juin-34	3,6775	0,3984	3,750	2,98	2,841	
26	1-juin-35	3,6613	0,3869	3,720	2,87		
27	1-juin-36	3,6440	0,3760	3,689	2,83		
28	1-juin-37	3,6270	0,3657	3,658	2,77		
29	1-juin-38	3,6100	0,3558	3,627	2,73		
30	1-juin-39	3,5933	0,3464	3,597			

[17] In the case of a spot transaction $DF_{ny} = 1$.

3.3.4 Calculation example of the 5 X 10 year par swap forward rate

The following Excel sheet, calculates the 5 year forward curve starting on 1st June 2014 and maturing on 1st June 2019. The resulting forward rate, 4.009%, is calculated according to the B.1.3.6 above formula based on the difference between the 5 and 10 year Discount Factor divided by the sum of the 6 to 10 Discount Factors.

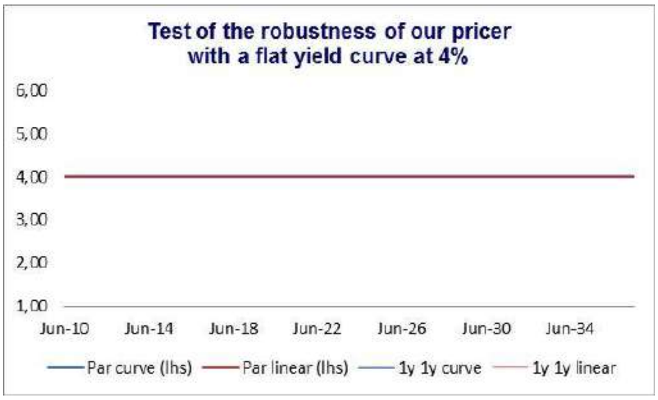
SOMME =(G17-G22)/(SOMME(G18:G22))*100

	D	E	F	G	H	I	J	K
11	6 m Euribor par and zero coupon swap curve, zero coupon & Forward curve							
12	Year	01/06/2009	Par curve	Discount Factor	Zero coupon	1 year fwd	5 year Fwd	year fwd
13	1	1-juin-10	1,8875	0,9815	1,888	2,34	3,106	3,576
14	2	1-juin-11	2,1130	0,9590	2,115	2,78	3,428	3,802
15	3	1-juin-12	2,3285	0,9331	2,335	3,23	3,684	3,990
16	4	1-juin-13	2,5450	0,9039	2,559	3,57	3,859	4,129
17	5	1-juin-14	2,7370	0,8727	=(G17-G22)/(SOMME(G18:G22))*100 = 4.009%			
18	6	1-juin-15	2,8885	0,8414	2,920	3,94	4,143	4,305
19	7	1-juin-16	3,0233	0,8095	3,065	4,03	4,256	4,347
20	8	1-juin-17	3,1335	0,7782	3,185	4,09	4,365	4,370
21	9	1-juin-18	3,2250	0,7476	3,285	4,32	4,461	4,371
22	10	1-juin-19	3,3168	0,7166	3,388	4,39	4,501	4,331
23	11	1-juin-20	3,3968	0,6865	3,479	4,51	4,505	4,270
24	12	1-juin-21	3,4705	0,6569	3,564	4,56	4,460	4,174

3.3.5 Testing the spread sheet used to calculate zero coupon and forward rates

In theory, with a totally flat par curve the zero coupon and forward rates must be equal to the par curve, since like in the actuarial world, the yearly coupons of the par curve can be reinvested at the same rate.

In the chart opposite we have applied a flat rate of 4% on each of the yearly steps of the par swap curve. It shows that the 1 and 10 year quartely forward rates that are derived from the par curve are stricly identical to 4%.



3.4 Interest rate swaps' fixed rate analytics, long term swaps' convexity

This paragraph briefly recalls that the fixed rate components of an interest swap, its fixed leg and the 1st fixed coupon of its floating leg, can be analyzed as a fixed rate bond. The standard analytics used in the 1st Volume to measure the sensitivity of the bond price to a change in market yields, are applicable to interest rate swaps, notably the duration, modified duration and convexity concepts.

3.4.1 The inverse correlation between fixed rate instruments' yields and swaptions' volatility

This paragraph focuses on the fixed leg convexity component of par interest rate swaps, since these derivative instruments are particularly suited to manage long term positions with reasonable credit risks.

- The price of any fixed rate instrument is exposed to a parallel shift of the yield curve, and the intensity of this price change increases with long maturity instruments.

- The modified duration is the 1st derivative of a fixed rate bond price; it is a linear measure of the change in the price of a bond in response to its change in yield to maturity.

- The convexity is the 2nd derivative of the price of a bond. It enables to measure the price variation of a fixed rate bond with more precision than the modified duration, especially in the case of large yield changes, since the relationship between yield and bond price is not linear. For large changes in market yields, the modified duration always understates the value of the bond; it underestimates the increase in bond price when the yield falls, and it overestimates the decline in price when the yield rises. The opposite chart illustrates the impact of convexity on price variations.

- The convexity of a bond increases with its maturity, as shown in the following table.

Maturity	10	15	20	30	50
Modified Duration	8,06	11,03	13,45	17,04	21,05
Convexity	0,80	1,54	2,37	4,11	7,12

- An investor having to maintain a fixed duration of its portfolio should better invest in a 50 year bond than in a 30 year bond. The 50 year bond price will drop less, in the case of rising long term rates and gain more, in the case of decreasing long term rates; a rise in volatility would benefit to the price of the most convex bond.

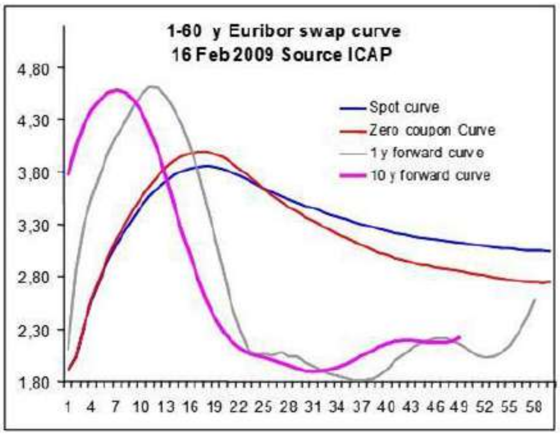
- However, fixed rate bonds do not have the same level of liquidity as interest rate swaps. And that is true even more nowadays

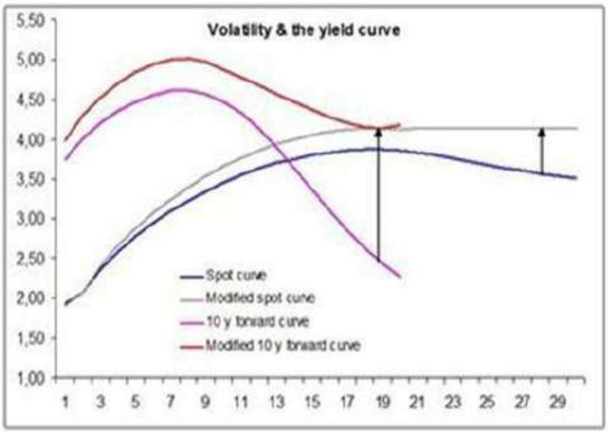
- For these reasons, fixed income instruments' prices incorporate a convexity component that is strongly inversely correlated with market rates volatility, as shown in the following chart. The left hand scale of this chart shows 10 year in 1 year swaptions' volatility and its right hand scale, the rate of 10 year swaps in reverse order.

3.4.2 Forward swaps’ convexity component

“In the early 80’s forward interest rates used to be calculated from the spot swap curve considered as a static *parameter* whose future evolution had no impact on the current spot curve. Today, by an iterative process, the swap spot curve is built according to the non arbitrage theory by using forward interest rates that have been corrected to incorporate the yield curve volatility”[18].

The two following charts illustrate forward swaps’ rates.





Many market participants believe that:

According to the Preferred Habitat theory a normal yield curve should slope upward to compensate for the risk of investing in long maturities.

And that in long maturities the forward rates should be flat, since it is very difficult to have a precise anticipation of the shape of the yield curve for the long term.

But if it were the case, traders could create very profitable convexity trades by borrowing for instance the 10 year in 10 year forward rate and lending the 10 year in 20 year forward rate that is much more convex than the 10 year forward. Against this gamma positive strategy a trader could sell 10 year in 20 year swaptions.

This explains why, when swaptions’ volatility trades at a very high level, the swap curve is inverted for very long maturities.

[18] A Brief History of Interest Rates Futures and Options World 1998, Supplement on the 25th Anniversary of the Publication of the Black-Scholes Model

4 Long term interest rate swaps floating leg, FRAs and Euribor futures financial relationship

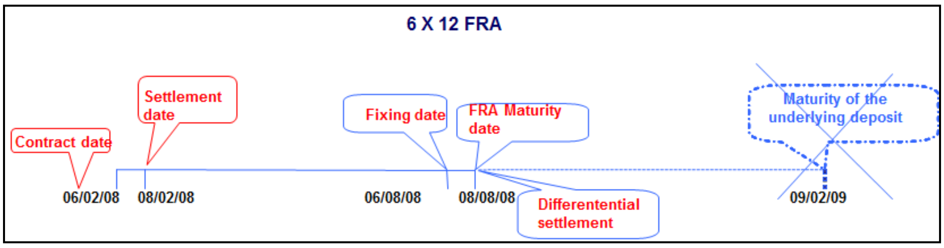
Simply speaking, a 10 year IRS referenced to 3 month Euribor can be analyzed as a string of FRAs bearing this 3 month Euribor reference and a 10 year IRS referenced to Eonia, as a string of 1 year Eonia forward swaps.

4.1 The relationship between IRSs and FRAs

4.1.1 FRA definition

A FRA is an Over The Counter (OTC) short term forward contract where the two counterparties agree to exchange at the maturity of this forward contract[19]:

- The discounted rate differential between the future rate agreed on the contract,
- And the spot fixing rate applying at maturity of the forward contract to a then spot money market deposit bearing a given maturity and a given notional amount specified in the contract. The table following illustrates a 6 X 12 FRA



4.1.2 FRA Standard conventions and utilization

- A 6 X 12 FRA corresponds to a future contract bearing a 6 month future period and a then 6 month notional deposit. In this example, the future contract settles on 08/02/2008 and matures on 08/08/2008
- FRAs are quoted in rate, as it is the standard convention on money market instruments, the buyer covers its market exposure to a rise in interest rates (borrowing exposition) and the seller to a decrease in money market rates (lending exposition).
- At the FRA maturity, the underlying deposit is not exchanged between the two counterparties: They only exchange the present value calculated in € between the contractual FRA rate and the Euribor fixing applied to the notional amount and maturity of the underlying deposit. If this differential is negative, meaning that spot market rates are lower than the contractual rate, the buyer of the FRA pays the differential to the seller and vice versa.
- FRAs are not traded on organized exchanges and their contractual terms are not standardized. FRAs that developed in the early 80s can be considered as the most basic interest rate derivative. Although not describing in details the features of FRAs, we expose paragraph ? how the financial crisis has largely transformed their valuation.

[19] i.e. the settlement date of the underlying money market deposit.

4.1.3 Calculation formula of the yield differential in € applicable on short term FRA maturities

The following formula applies to the calculation, at the FRA maturity, of the discounted value of the € yield differential exchanged between the two counterparties on money market FRAs, bearing a total maturity, the forward contract maturity plus the notional maturity of the underlying deposit, inferior or equal to 1 year[20]. This discounted value is calculated at maturity of the FRA at the Euribor fixing rate fixed two business days before the maturity of the FRA:

$$FRA_{Dif} = \frac{(Fixing_{Spot} - FRA_{Rate}) * Not_{am} * nd_{Underlying} /360)}{(1 + Fixing_{Spot} * nd_{Underlying} /360\,)} \qquad (B.1.3.5)$$

With

FRA_{Dif}	*: Discounted yield differential paid at the FRA maturity*
$Fixing_{spot}$	*: Euribor or Libor spot fixing*
FRA_{Rate}	*: FRA contractual rate*
$NOAT_{am}$	*: Derivative notional amount*
$ND_{Underlying}$	*: Number of days in the underlying period*

Example, On 6 Feb 2008 two counterparties agree on the 6 X 12 following FRA:

° Notional amount	: 100 M
° FRA Contractual Rate	: 3.76%
° FRA Settlement Date	: 8 Feb 2008
° FRA Maturity Date	: 8 Aug 2008
° Underlying Deposit Maturity Date	: 8 Feb 2009
° Spot Fixing 6 M Euribor Underlying reference on 6 Aug 2008	: 3.32%
° € yield differential at the FRA maturity	:

FRA_{Dif} = (($Fixing_{Spot}$ – FRA_{Rate}) * Not_{am}* $ND_{Underlying}$/ 360)/ (1 + $Fixing_{Spot}$ * $ND_{Underlying}$ / 360)=

((3.32%-3.76%)*100 M*184/360)/(1+3.32%*184/360)=- € 221 136.45

Since the fixing rate is lower than the FRA contractual rate, the buyer pays to the seller € 221 136.45.

4.1.4 Valuation of short term FRAs

This paragraph exposes the traditional methodology of pricing short term FRAs by using money market rates. We will later on explain how the pricing of these instruments has evolved since August 2007.

Initially in theory, in order to hedge the sale of a 6 X 12 FRA, a bank had to realize in the interbank market the following operations:

- To borrow from the interbank market a 12 months loan, on the maturity of the FRA underlying deposit,
- And to lend to the interbank market a 6 month deposit, on the 6 month maturity date of the FRA.

The formula following is used to calculate the rate of short term FRAs:

[20] We have detailed paragraph ? the calculation of long date forward rates

$$FRA_{RAte} = \left(\left(\frac{1 + LD_{Rate} * LD_{nd} \ / 360}{1 + SD_{Rate} * SD_{nd} \ / 360} \right) - 1 \right) * \left(\frac{360}{LD_{nd} - SD_{nd}} \right) \quad (B.1.3.6)$$

With

FRA_{Rate}	: FRA contractual rate
LD_{Rate}	: Long deposit rate
SD_{Rate}	: Short deposit rate
LD_{nd}	: Long deposit number of days
SD_{nd}	: Short deposit number of days

Example of calculation of a 6 X 12 FRA in €

° 6 m deposit rate:	4.0%
° 6 m deposit number of days:	182
° 12 m deposit rate:	4.0%
° 12 m deposit number of days:	182
° 6 X 12 FRA rate:	**3.9207%**

3.9207% = (((1+ 4.0%*367/360)/(1+ 4.0%*182/360))-1)* 360/185

When we described the long term yearly forward rate derived from the zero coupon curve, we indicated that in the case of a totally flat par swap curve, the zero coupon rates and the forward rates are identical to the rate of the flat par curve. For instance, if the 1 year par swap rate is at 4.0% and if the 2 year par swap curve is also at 4.0%, then the 1 year in 1 year forward rate is equal to 4.0%.

But as shown on the example above, on short term rates that is not the case:

- Long term interest rates in € are expressed as yearly actuarial rates bearing an Actual/ Actual basis on bond and a 30/ 360 basis on swaps which on a yearly coupon periodicity is equivalent,

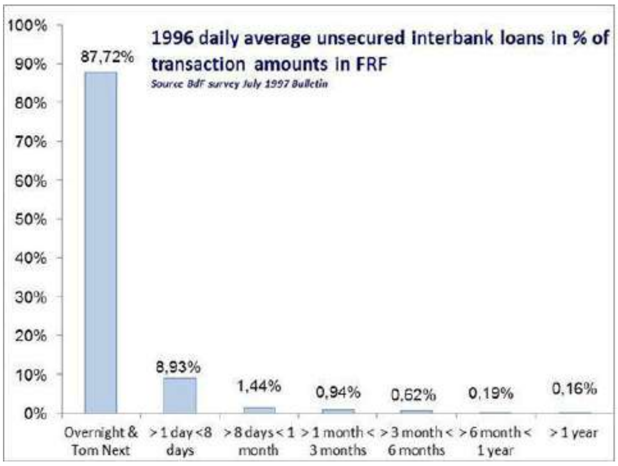
- Short term interest rate in € are expressed as a yearly simple yield paid at maturity of the instrument and they bear an Actual/ 360 basis. Therefore, the simple yield of a six month deposit bearing a 182 days maturity is higher, when expressed in actuarial terms, than the actuarial rate of a 12 month deposit, explaining the 3.9207% FRA rate found in the above example.

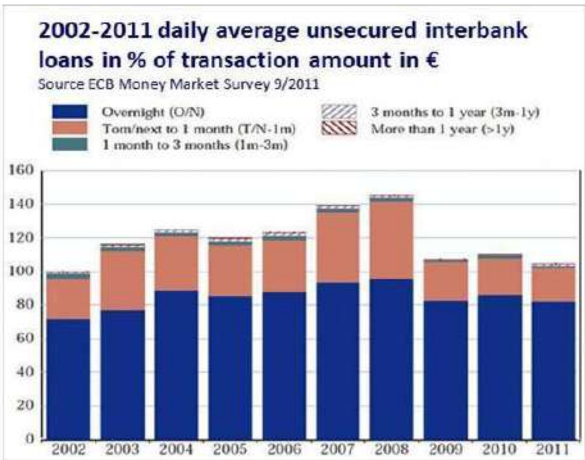
4.1.5 FRAs' valuation had already changed before the subprime crisis

The rate of the 1st FRA contracts launched at the beginning of the 80s were calculated by using money market deposits rates, since their Libor, or subsequently Euribor, fixing references were defined as a money market offered rate.

However this pricing methodology has rapidly changed, due to the following factors:

- The well recognized market illiquidity of interbank deposits bearing a maturity greater than 1 month: This admission is not recent. In its July 1997 Monthly Bulletin[21] the Banque de France indicated that less than 2% of the daily traded unsecured interbank loans had a maturity greater than 1month, cf. left chart following. The Sep 2011 ECB Money market survey published with a base index 2002 = 100, confirms that this illiquidity still remains, cf. right chart following[22].





- Since the introduction of the Basle ratios, interest rate derivatives like FRAs have expanded in order to reduce banks' balance sheets. Consequently, the use of "on balance sheet" interbank unsecured deposits to hedge "off balance sheet" contracts was not appropriate.

- For these reasons, up to October 2007, the banks usually hedged FRA transactions based on Euribor reference by using 3 or 6 months Eonia swaps and adding the 6-7 bps spread that prevailed on the market from Jan 1999 to Aug 2007.

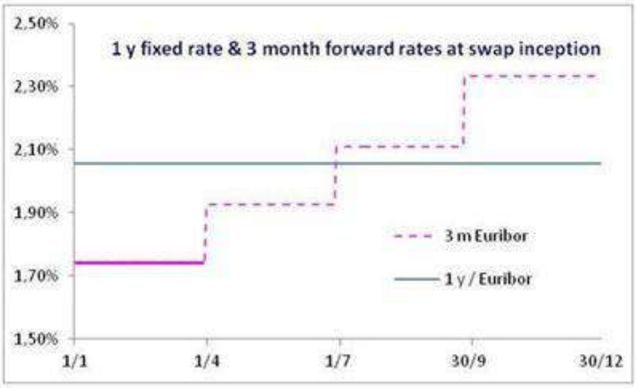
[21] L'analyse des flux d'opérations sur le marché monétaire français, Bulletin n° 44, July 1997, Jean-Marc Serrot, Frédéric Wilhelm :
http://www.banque-france.fr/archipel/publications/bdf_bm/etudes_bdf_bm/bdf_bm_44_etu_6.pdf
[22] http://www.ecb.int/pub/pdf/other/euromoneymarketsurvey201109en.pdf

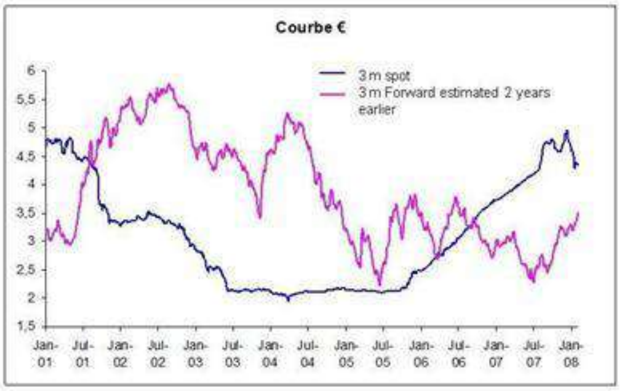
4.1.6 Short term IRS/ FRA Euribor relationship

When discussing in details the valuation of long term interest swaps, we have explained that:

- The fixed leg yield of a primary par swap, a not yet transacted swap, is equal to the "anticipated" compounded yield of its floating leg, since the yield of its "anticipated" forward rates is derived from the yield of the fixed leg. The same reasoning applies to short term IRSs,

- The 1st chart below shows the expected cash flows of a 1 year against 3 month Euribor swap at its 1st 3 month Euribor fixing date and the subsequent 3 month forward rates. In a steep yield curve environment, the expected values of the non yet fixed 3 month references is priced above the 1st 3 month Euribor fixing and also above the 1 year fixed rate, in the case of the two last floating coupon periods of the swap. Simulating the forward curve helps the traders to take position on the market. Very often the forward curve, especially in a steep curve environment, can be considered as exaggerated, as shown on the 2nd chart that historically compares 3 month spot fixings with the 3 m forward curve calculated 2 years earlier.





The value of the 3 month forward rates shown on the left chart remains indicative, these subsequent reference being fixed on their respective fixing date at the then market conditions. And the mark to market of a secondary swap depends only on the change in rates applicable to its fixed leg and 1st floating coupon.

For the valuation of a secondary swap, a trader mimics the rates of a "reversal" swap, a primary swap that bears the same maturity date and the same floating reference as the ones of this secondary swap. In the case of a secondary fixed rate receiver swap, this reversal swap will mimic a primary payer swap. The broken date of this primary reversal swap will be set as its 1st coupon period and all subsequent Euribor floating periods of these two primary and secondary swaps will be fixed on the same fixing dates. Therefore, the valuation of these swaps' floating leg is based on the sole change in the yield of the secondary swap fixed leg and of its 1st floating coupon leg, since the subsequent floating rate fixings applicable to this secondary swap and this reversal swap will be neutralized.

4.1.7 IRSs and Euribor futures contracts' relationship

We do not detail all the financial characteristics of Euribor future contracts, but briefly describe their main characteristics and indicate why, due to a convexity adjustment, FRAs and Euribor futures based on similar maturity dates do not bear, the same yield.

Definition: A Euribor future contract is a standardized 3 month FRA quoted on Liffe and Eurex exchanges. Long term bond futures' contracts being also traded on exchanges, money market futures have adopted some of the bond market conventions. Like bond futures, money market futures' rates are quoted in price: Their price, expressed in percentage, is equal to 100 minus their money market rate expressed as a yearly simple yield. For instance the rate of a 3 month Euribor futures' contract priced at 97.45 corresponds to a money market yield of 2.55%.

The inverse Euribor futures language: Euribor future quotations are expressed in price according to long term bond market conventions, contrary to the FRA money market conventions that are expressed in rate:

- Paragraph 1.4.1.2, we have explained that since FRAs are quoted in rate, the **buyer** covers its market exposure to a rise in interest rates (borrowing exposition) and the seller, to a decrease in money market rates (lending exposition).

- The Futures' money market language is inversed: Euribor futures are quoted in price, the **seller** covers its exposure to a rise in interest rates and the buyer, to a decrease in money market rates.

Euribor futures' main specifications

The table following briefly details the main contract specifications of Euribor future contracts quoted by Nyse Euronext on the Liffe[23].

Three Month Euro (EURIBOR) Interest Rate Futures	
Unit of Trading	€1,000,000
Delivery Months	March, June, September, December, and four serial months, such that 25 delivery months are available for trading, with the nearest six delivery months being consecutive calendar months
Quotation	100.00 minus rate of interest
Minimum Price Movement (Tick Size and Value)	0.005 (€12.50)
Last Trading Day	10.00 Two business days prior to the third Wednesday of the delivery month
Delivery Day	First business day after the Last Trading Day
Trading Hours	01.00 – 21.00

[23] Source Liffe Short Term Interest Rate Products: http://globalderivatives.nyx.com/sites/globalderivatives.nyx.com/files/268469.pdf

And the Bloomberg page following illustrates the Liffe Euribor quotations on 2/12/2008


<HELP> explications. ComdtyCT
<GO> pr faire défiler contrats. EXCH affiche les marchés en temps réel.
Tableau des contrats
3MO EURO EURIBOR Surveillance en diff. activée
Page Web de Bourse Date Pricing: 2/12/08 Afficher Prix: 2
EURONEXT.LIFFE Prix différés
Date en gris = options traitées

Défiler		Dern.	Var. %	Heure	+Haut	+Bas	P.O.	VolTot	Clôture précdte
							4160412	877813	
1) ERG8	Feb08	95.670d	unch	15:42	95.675	95.600	60198	2999	95.670
2) ERH8	Mar08	95.750d	-.02%	15:54	95.765	95.735	812471	101830	95.765
3) ERJ8	Apr08						0	0	95.885
4) ERK8	May08						0	0	96.035
5) ERM8	Jun08	96.215d	-.02%	15:54	96.255	96.190	724847	156986	96.235
6) ERN8	Jul08						0	0	96.285
7) ERU8	Sep08	96.510	-.05%	15:55	96.580	96.480	578994	185132	96.555
8) ERZ8	Dec08	96.645	-.05%	15:55	96.720	96.615	543038	158611	96.690
9) ERH9	Mar09	96.695	-.04%	15:55	96.770	96.665	408531	102433	96.735
10) ERM9	Jun09	96.650	-.04%	15:55	96.715	96.615	296458	79390	96.685
11) ERU9	Sep09	96.560	-.04%	15:55	96.630	96.525	240068	42119	96.600
12) ERZ9	Dec09	96.445d	-.04%	15:54	96.505	96.410	221325	28942	96.485
13) ERH0	Mar10	96.350d	-.05%	15:48	96.410	96.320	97546	11091	96.395
14) ERM0	Jun10	96.260d	-.05%	15:48	96.320	96.230	54827	4952	96.310
15) ERU0	Sep10	96.195d	-.05%	15:29	96.245	96.165	55148	1826	96.240
16) ERZ0	Dec10	96.125d	-.05%	15:17	96.170	96.085	43714	988	96.170
17) ERH1	Mar11	96.120d	-.02%	14:00	96.120	96.115	10936	514	96.135

Australia 61 2 9777 8600 Brazil 5511 3048 4500 Europe 44 20 7330 7500 Germany 49 69 9204 1210 Hong Kong 852 2977 6000
Japan 81 3 3201 8900 Singapore 65 6212 1000 U.S. 1 212 318 2000 Copyright 2008 Bloomberg Finance L.P.
G434-29-1 12-Feb-08 16:11:04

4.1.8 The FRA/ Euribor futures' convexity adjustment:

Euribor future prices include a convexity adjustment due to the differences of pricing interest rate changes between FRAs and Euribor Futures:

- The € yield differential exchanged at the FRA maturity, is discounted at the Euribor fixing rate fixed two business days before the maturity of the FRA. Therefore, for a given discount rate, the net present value of this differential decreases, when applied to long forward underlying deposits, for instance 12 months instead of 3 months. Like with a bond, the price of a FRA is convex[24]. The relationship between the change in yield and the change in price of a fixed rate bond is not linear.

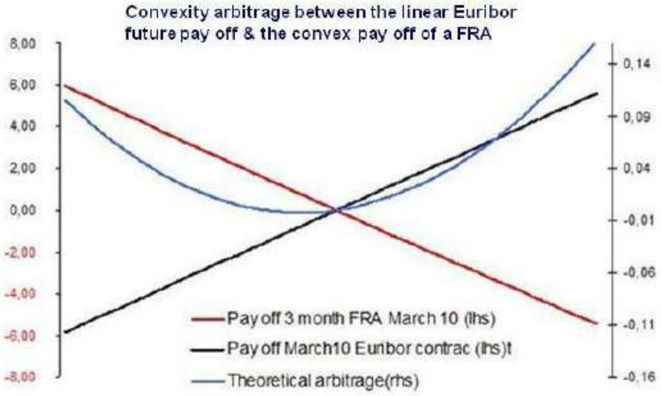
- Future contracts traded on an exchange have to be largely standardized, in order to improve their liquidity. For this reason, on a Euribor future contract, a 0.005 change in price corresponds to a daily margin call of € 12.5, on a € 1 million contract[25]. And the margin calls paid to or received from the exchange are not discounted over the life of the underlying deposit. A ½ bp price change on a Euribor future contract is always worth € 12.5, whatever the maturity of the contract. Therefore the change in price of Euribor future contracts is linear.

[24] As exposed Volume A paragraph 2.2.8.4, the convexity of a fixed rate bond is equal to the second derivative of its price in relation to its yield divided by its price.
[25] Calculated on a € 1 000 000 contract on a 90/360 basis, the minimum price change, ½ bp, equals € 12,50.

As a result of these differences, Euribor futures market rates are always priced higher than the ones of FRA contracts bearing the same maturity. This phenomenon is called the convexity bias of Euribor futures prices.

The chart following illustrates the potential convexity arbitrage in December 2008 between a March 2010 Euribor short future contract hedged by a 3 month short FRA:

- In the futures language, contracts are expressed in price and a short future position corresponds in fact in the FRA language to a long rate position.
- Since the Euribor future contract pays a linear payoff, a trader will always benefit from this asymmetric position in a volatile interest rate environment. In the case of a rise in interest rates, this trader will always benefit from the higher linear payoff generated by its short future Euribor position, while benefiting in the case of a decrease in interest rate of the convex price change of its short FRA position.
- Consequently, futures market rates are always higher than forward rates by a few bps, incorporating in their price the anticipated volatility of interest rates.



4.2 Historical Euribor and Eonia fixings' relationship

Euribor and Eonia are the main unsecured money market references in €.

This paragraph concentrates on the description of the impact of the subprime crisis on € money markets and especially on Euribor fixings, the main floating reference used in €, Euro Libor references published by the BBA being barely used. At the same time, in other international currencies like USD, GBP or JPY, Libor references, considered as the international benchmark money market floating references, have also been similarly affected. Overall money market spreads between Euribor or Libor fixings and overnight rates have considerably widened.

During the 8 years that followed the introduction of the euro, Euribor and Eonia swap curves used to trade at a spread of around 6-7 bps. However since the subprime crisis, this spread has drastically changed: In August 2007, interbank money markets' fixings have shown one of the first financial symptoms of the subprime crisis, with the tremendous divergence between the rates of the unsecured Euribor or Libor fixings and the ones of same maturity Overnight Index Swaps, OIS swaps.

Since IRSs fixed and floating legs' yield have to be identical, the large money market spreads experienced on their swap floating leg references have been immediately translated into the fixed leg of short and long term IRSs, strongly distorting the prices of all fixed income instruments and creating a large segmentation between IRS curves bearing different floating references.

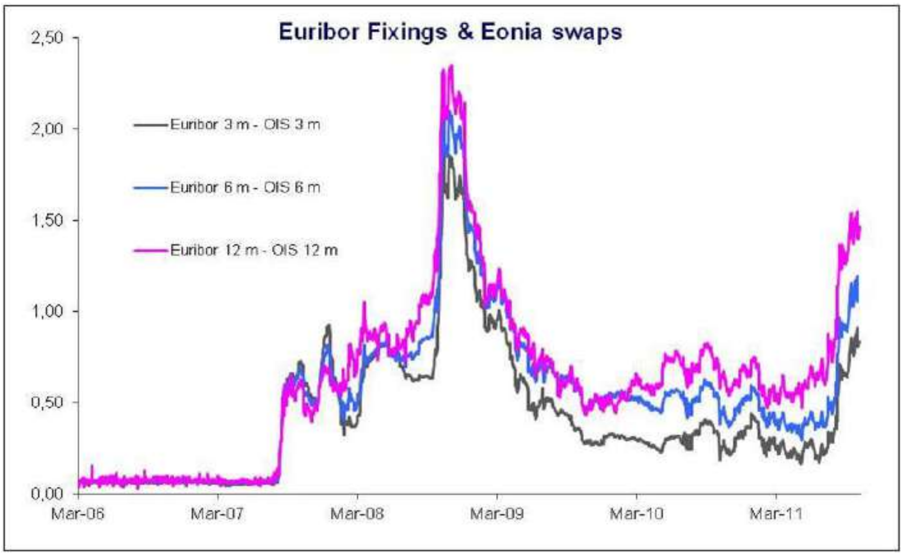
4.2.1 Euribor fixings and same maturity Eonia swaps' spread

The Euribor / Eonia spread that have been largely in focus with the emergence of the subprime crisis can be expressed in different manners:

- In a very basic manner, this spread deducts from the value of Euribor fixings, bearing for instance a maturity of 3 or 6 months, the value of the last published Eonia fixing. However with a steep yield curve, 6 month Euribor fixes at a higher level than 3 month Euribor. Moreover, daily Eonia fixings can be rather volatile, notably at the end of the reserves' periods.

- A more precise calculation of this spread deducts from the Euribor fixing maturing in *n* months the rate of the OIS swap bearing a similar *n* maturity.

This paper refers to the "Euribor Eonia spread" as the difference between Euribor (or Libor fixings) as the difference between these 3 or 6 months fixings, for instance, and same maturity OIS swaps. The chart following illustrates these spreads' striking changes that have occurred in €, since August 2007.



4.2.2 The theoretical 6-7 bps value of the Euribor Eonia swap spread before August 2007

From 01/1999 to 07/2007, as shown on the chart above, same maturity fixings and Eonia swap spreads had been trading at around 6-7 bps, nearly independently of the maturity of their floating reference[26]:

- Both Euribor and Eonia fixings apply to unsecured interbank deposits and they are published by the European Banking Federation, EBF; these two references are based on the contributions of the same 44 banks panel banks[27].

- Apart from the maturity of their reference, 1 day for the Eonia, 3, 6 or 12 months for the Euribor, the main difference between these two references corresponds to their definition: Euribor is an offered rate and Eonia, a transacted one.

[26] Before August 2007, the 6 month Euribor swap curve traded at around 1 bp. above the 3 month curve, which can nowadays be considered as negligible.
[27] Number of panel banks in June 2011.

- Under normal market conditions up to August 2007, interbank bid offer spreads used to be set at 1/8%, i.e. 12.5 bps.

- Since Eonia is a transacted rate, it was fixed on average at the middle of the bid offer spread, i.e. around 6.5 bps below the offered rate.

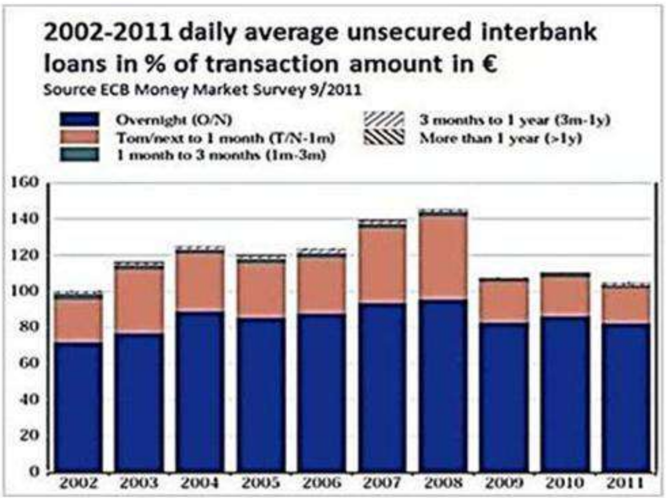
4.2.3 For many years, interbank unsecured transactions had concentrated on short maturities

Before August 2007, the historical 6-7 bps value of this spread corresponded to the sole theoretical difference between the Euribor offered rate and the Eonia transacted rate: Banks were not willing to pay an additional liquidity spread on long dated Euribor fixings, since for more than 10 years, pure interbank unsecured money markets' transactions, bank to bank transactions, had concentrated on short maturities.

The already shown chart below[28] published in the September 2011 ECB money market survey confirms that:

- On the daily unsecured interbank market, 70% of the volumes trade overnight,

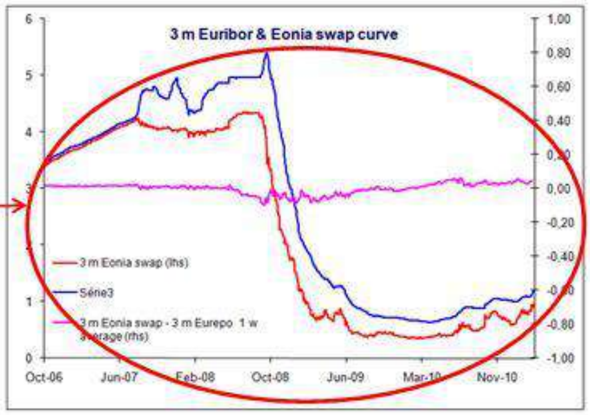
- And maturities below one month account for more than 95% of the daily traded volumes.



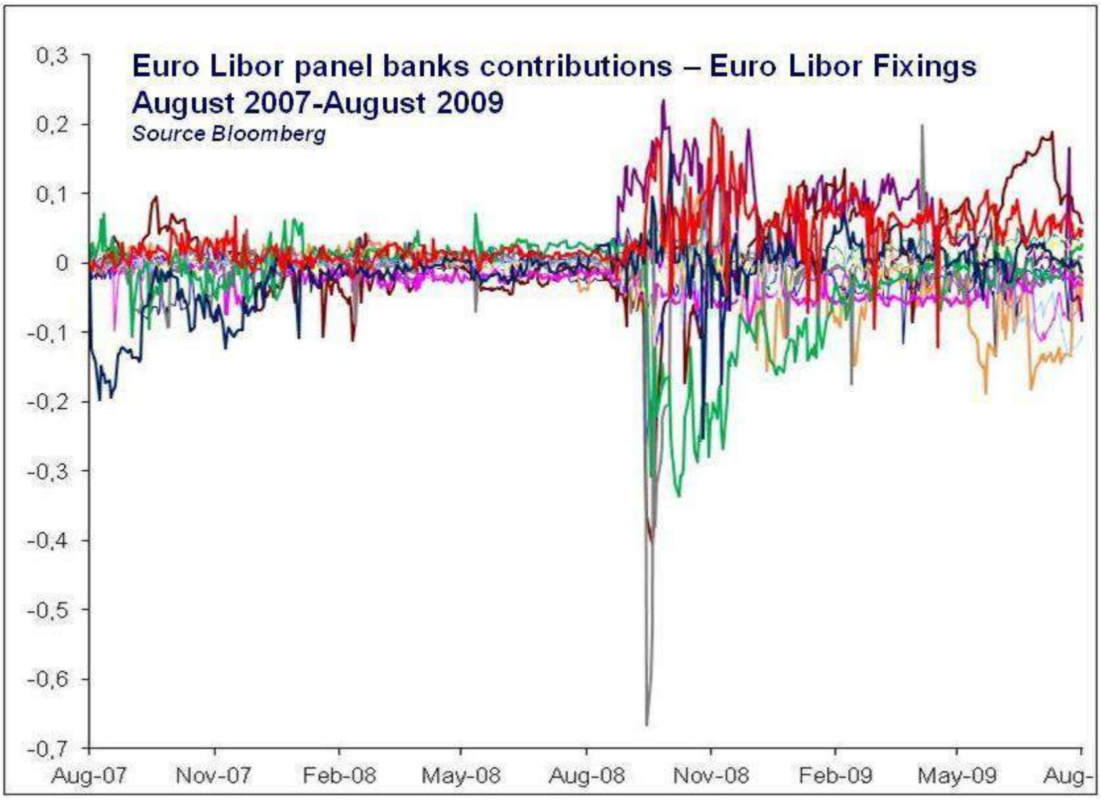
[28] Cf. paragraph 1.4.1.5

4.3 The impact of the subprime crisis on the Euribor and Libor contributions

The two charts following illustrate the change in the Euribor OIS swap spread on a 3 month maturity, since August 2007.



The chart below shows the divergence between individual Euro Libor panel banks contributions and Euro Libor fixings from August 2007 to August 2009. We use the Euro Libor contributions, the daily individual Euribor contributions being only available since September 2008. However, current published Euribor contributions compared with the Euribor fixings show a very similar pattern.



As seen on this chart that starts in August 2007, the dispersion of the Libor 16 panel banks' contributions has been considerably amplified with Lehman Brothers' failure. So as to analyze these contributions, we have classified these banks in two main categories, the 5 banks that only contribute to the Libor fixings and the 11 banks that have also contributed 1 hour earlier to the Euribor fixing:

- Before 2009, the 5 banks that only participate to the Libor fixing used to quote a rate quite close to the already published Euribor fixing. We will describe later the general pricing changes that have occurred in 2009, due to the "amplification" of the BBA Libor definition.

- During the August 2007-August 2009 period, the 11 banks that had also participated to the 1 hour earlier Euribor fixing generally quoted a Libor rate, either higher or lower than the subsequent Libor fixing that can be considered as coherent with their Euribor declarations. It can be noticed that one the large international investment contributing to the both Euribor and Libor panels declared a rate very close to the subsequent fixing, cf. purple line. To the contrary, some of these 11 banks contributed a much lower rate than the other banks and their quotations were most of the time excluded from the average calculation[29].

- Since the inclusion of the certificates of deposits' rates in the calculation of Libor fixings, all these banks have lowered their contributions by around 6-8 bps.

The reason why some large banks during this period have nearly constantly contributed such a low rate can be surprising, since Libor and Euribor fixings have been originally created to serve as an official reference for syndicated loans. However, as we will later see, Libor and Euribor references bear a subtle difference: According to these fixings' respective definitions, Libor banks have to contribute the rate at which they could borrow on the interbank market, where Euribor banks have to estimate the potential cost of funding of a prime bank.

On this subject, in its March 2008 quarterly review, the BIS concluded that even if "a few banks engaged in manipulative behaviour, then the trimming procedure ensured that their rates were not used to calculate the rate fixing. If a majority of banks engaged in strategic behaviour, then trimming alone would not have mitigated the impact on the fixing. That said, there is little evidence that this was the case".

The following table details the composition of Libor and Euribor panel banks in March 2011.

Libor euro panel banks
Abbey National plc
Bank of Tokyo-Mitsubishi
Barclays Bank plc
Citibank NA
Credit Suisse
Deutsche Bank AG
HSBC
JP Morgan Chase
Lloyds Banking Group
Mizuho Corporate Bank
Rabobank
Royal Bank of Canada
Société Générale
The Royal Bank of Scotland
UBS AG
West LB AG

Euribor panel banks		
Allied Irish	Citibank	MPSI
Banca Intesa	Commerzbank	National Bank of Greece
Banque Postale	Danske Bank	NATIXIS
Barclays	Deutsche	Norddeutsche
Baylaba	Dexia	Nordea
BBVA	DZ Bank	Pohjola Bank
BCEE	Erste	Rabo
BNPP	HELE	RBS
BoIreland	HSBC	RZB
BSCH	ING	Soc Gen
BTMU	JPM	Svenska
Calyon	KBC	UBS
CECA	La Caixa	Unicredito
CGD	LBB Berlin	WLB
CIC	LBBW	

Grey cells indicates banks contributing to both Libor and Euribor panel in March 2011

[29] According to the Libor trimming rule, the daily 4 highest and 4 lowest contributions are excluded before averaging the contributions of the 8 remaining banks.

4.4 Libor and Euribor definitions

4.4.1 Libor

Considering the changes in course due to the “Liborgate” to the Libor definition and practices, the CNO-French Banking Association intends to update as soon as possible the current chapter on Libor definition, once the issue is stabilized.

Indeed, following the issue of the Wheatley Review of Libor, the BBA Council has indicated it would support any recommendation that responsibility for Libor should be passed to a new sponsor, in the view to ensuring the provision of a reliable benchmark.

The Weatley Review concluded that ‘it now falls to the Government, the BBA, the banks and other market participants, and the international regulatory community, to consider (these) recommendations’.

More information can be found on current definitions and practices regarding the Libor on the Official Libor website.

4.4.2 The Euribor definition

The provisions of the Euribor Code of Conduct read as follows: “Panel banks must quote the required euro rates. To the best of their knowledge, these rates are the rates at which euro interbank term deposits are being offered within the EMU zone by one prime bank to another at 11:00 am Brussels time (“the best price between the best banks”)”.

The Code of Conduct adds that panels banks must quote the required euro rates:

- for the complete range of maturities as indicated by the Steering Committee,
- on time as indicated by the screen provider,
- daily, except on Saturdays, Sundays and Target holidays,
- accurately with two digits behind the comma.

More details can be found on the official website of Euribor, a non-for profit organization: www.euribor-ebf.eu.

Euribor fixing is based on contributions from 44 different banks (July 2012), who are active market players. The six highest and the six lowest contributions are systematically excluded from the calculation each day.

Publication of the rates is done by Thomson Reuters.

4.5 Euribor spreads

4.5.1 Credit and liquidity components of the Euribor Eonia swap spread

Since the onset of the financial turmoil, the presence of a liquidity premium incorporated in the rates of Euribor fixings is not contested. However, the concept of financial liquidity is not always easy to define and it is not directly possible to price separately the value of this liquidity premium and that of its credit risk.

4.5.1.1 The concept of financial liquidity is not always easy to define…

Bank of England description of liquidity risk

Referring to a BIS paper, "The management of liquidity risk in financial groups", published in May 2006 by the joint Forum of the Basel Committee on Banking Supervision[30], the Bank of England describes two types of liquidity risks:

- "Funding liquidity risk occurs if a firm is not able to meet its cash-flow needs;
- Market liquidity risk materializes if a firm cannot easily offset or eliminate a position without significantly affecting the market price.

These two concepts can be linked.

- A firm facing funding liquidity risk may need to sell assets to meet cash-flow needs.
- But if asset markets are relatively illiquid, then the firm may be forced to sell them at a low price. In extreme events, feedback loops between the two may be generated. An initial fall in asset prices might trigger further asset sales, for example, to meet margin calls or because risk limits have been breached. Prices could then be driven down further and so on"[31].

4.5.1.2 … It is difficult to create a reliable liquidity index and to price a liquidity premium

Several indicators confirm that the price of various fixed income instruments should bear a liquidity premium on top of their credit spread:

- As explained Volume A, two different bonds launched by a same issuer on a same maturity can bear a different asset swap margin.
- In a July 2011 preliminary paper, Alain Monfort and Jean-Paul Renne propose a model for disentangling credit from liquidity risks in fixed income markets, based on the liquidity factor identified in the KFW-Bund spreads[32].

In the following paragraphs we illustrate the changes in prices of money market instruments since August 2007. This suggests that, on top a credit spread, Euribor fixings now incorporate a liquidity premium due to the sudden and huge liquidity needs encountered by many banks in the money market:

- Like with the anticipated inflation and the inflation risk premium of inflation linked bonds described in the 1st volume, we do not seek to distinguish the precise value of each of these two components;

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[30] http://www.bis.org/publ/joint16.pdf
[31] http://www.bankofengland.co.uk/publications/fsr/2007/fsrfull0704.pdf
[32] This model identifies a liquidity-related pricing factor by exploiting the term structure of the KFW-Bund spreads. Bonds issued by KFW are explicitly guaranteed by the Federal Republic of Germany and benefit from the same credit quality as German Bunds, but they are less liquid. This study indicates in its conclusion that the KFW bund spread on a same maturity is essentially driven by liquidity components. Credit and liquidity risks in euro-area Sovereign yield curves, Alain Monfort and Jean-Paul Renne, http://www.ieseg.fr/files/2011/09/Renne.pdf

- However we hope that the market changes described in these following paragraphs will help the readers to better understand the management of interest rate exposure.

4.5.1.3 Illustration of the credit and liquidity components of Euribor/ Eonia swap spreads

We describe the changes occurred in money markets over the last four years and suggest that:

- From now on, and contrary to the pre-subprime crisis, a liquidity premium is attached to Euribor fixings credit spreads.

- And that the value of this "liquidity premium" incorporated in money market fixings should be linked with the maturity length of their floating references.

To illustrate these credit and liquidity components of Same Maturity Euribor/ Eonia swap spreads, we use the concept of the Relative Yield Differential between different Euribor/ Eonia swap spreads.

4.5.1.4 The relative yield differential between Euribor/ Eonia swap spreads

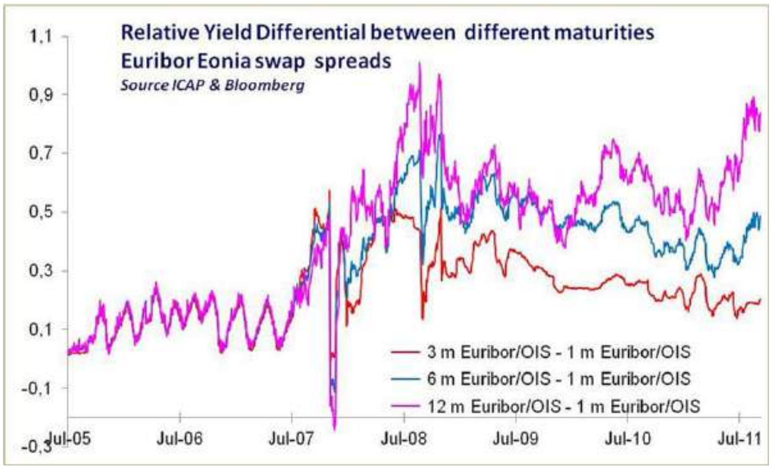
The chart below compares what we call **the Relative Yield Differential, *RYD*,** between:

1) Same Maturity Euribor fixing/ Eonia swap spread on one hand, for example the 3, 6 or 12 months,

2) And Same Maturity 1 month Euribor Fixing/ Eonia swap spread on the other hand.

For example, the (6m – 1m) Relative Yield differential is calculated according the following formula:

$$RYD_{6m\text{-}1m} = (Euribor_{6m} - Eonia_{6m}) - (Euribor_{1m} - Eonia_{1m}) \quad (B.2.2.4\ )$$



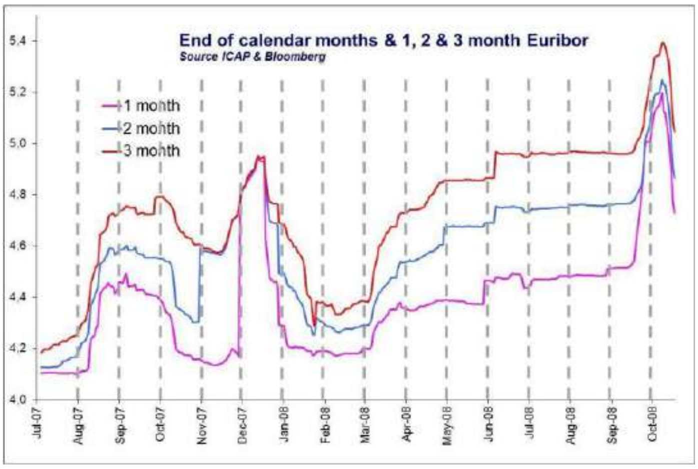
These relative yield differentials compare credit spreads and liquidity premium on different maturities:

- Before August 2007, banks were apparently not willing to pay a liquidity premium in order to secure long maturity funding: The historical 6-7 bps spread between same maturity Euribor fixings and Eonia swaps corresponded to the sole theoretical difference between the Euribor offered rate and the Eonia transacted rate. And this 6-7 bps spread was applied on any fixing from 1 month to 1 year.

- Since August 2007, Euribor fixings incorporate a liquidity and a risk premium. At the end of November 2007, as shown on the chart of the previous page, the same maturity 1 month Euribor fixing/ Eonia swap spread became more expensive than other longer same maturity Euribor Eonia spreads.

  Consequently, the (3 m – 1 m) Relative Yield Differential inverted. This sudden change can be easily understood: At the end of November, the maturity of the 1 month fixing fell into the next calendar year. This behavior suggests that banks were either concerned by the liquidity of the interbank market at the end of the fiscal year or by some window dressing considerations.

- The chart below illustrates since August 2007, the behavior of same maturity Euribor fixings and Eonia swap spreads, at the beginning of a new calendar month.



Without trying to price the respective values of the credit spread and the liquidity spread incorporated into those fixings, we analyze the relationship between Euribor and Eonia, on one hand, and Eurepo and Eonia, on the other hand.

4.5.1.5 Analysis of the credit and liquidity components of the Euribor Eonia swap spread

As shown on the chart above, the spreads between same maturity Euribor fixings and Eonia swaps are quite volatile, notably at changes of calendar months.

Euribor and Eonia fixings are contributed by the same panel of 44 banks[33] and, in theory according to their definitions, they should price the same interbank counterparty risk. However, the crisis created some differential incredit risks among banks. As Eonia and derived OIS (overnight interest swaps) are almost risk free, while Euribor transactions, that usually are 1/3/6 months matured, are not, the OIS/Euribor spread is mainly reflecting the level of the credit risk, or of mutual trust, in the interbank market.

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[33] The 44 banks, in July 2012, contributing to these fixings must quote:
- For the Euribor®, the rate at which euro interbank term deposits are being offered by one prime bank to another within the EMU zone.
- And for the Eonia®, the rate of their daily overnight unsecured lending transactions in the interbank market, initiated within the euro area.

4.6 The Eurepo

According to Euribor-EBF, Eurepo is the rate at which one prime bank offers funds in euro to another prime bank, if in exchange the former receives from the latter Eurepo General Collateral, GC[34].

The current Euribor Eurepo spread gives another illustration of the increased importance of liquidity.

Sponsored by the European Banking Federation, the Eurepo is the rate "at which at 11:00 am Brussels time, one bank offers, in the euro-zone and worldwide, founds in euro the another bank if in exchange the former receives from the latter the best collateral within the most actively traded European repo market".

Eurepo is published by Thomson Reuters.

The Collateral is the General Collateral, whose list is fixed by the Steering Committee of Euribor EBF.

Eurepo GL currently consists of any or all of Euro zone government guaranteed bonds and bills.

4.7 Interest rate swaps credit risk

Interest rate swaps' credit risk has two components, its counterparty risk and the credit risk associated with its reference:

- **The swap counterparty risk** over the life of the interest rate swap, for instance 10 years: Since pure interest rate swaps' notional amount is never exchanged, the sole counterparty credit risk of this swap transaction lies on the future evolution of interest rates. A bank having hedged its interest rate market exposure by using an OTC swap will lose its protection, in the case of the failure of its counterparty. However, it will only incur a loss if, due to adverse interest rate conditions, the failed counterparty owes it some money at the time of its failure. However this counterparty risk is in fact strongly reduced due to the collateralization of IRSs [35] [36],

- **The credit risk associated to the IRS floating rate reference,** for instance the 6 month Euribor money market underlying credit exposure. This counterparty risk is limited to the maturity of the floating reference since, on each fixing date, the panel contributing to Euribor fixings only includes banks that have not defaulted. Furthermore, Euribor fixing trimming rules exclude the 15% panel bank highest contributions and the 15% lowest ones. As indicated by Euribor-EBF, the sponsor of Euribor fixings, "the … banks quoting the Euribor… are of first class market standing and they have been selected to ensure that the diversity of the euro money market is adequately reflected, thereby making Euribor an efficient and representative benchmark"[37].

- However, from a pure theoretical point of view, the Eonia reference can be a volatile, in crisis times. As shown on the chart following, in October 2011, 3 m Eurepo rates and Eonia swaps started diverging, illustrating the flight to quality experienced in the Geman bund market.

[34] Eurepo General Collateral mainly consists of well rated debt securities such as government guaranteed bonds and bills of European countries.

[35] http://www.risk.net/risk-magazine/feature/1594823/the-price-wrong

[36] http://www.interestrateswaps.info/Swap%20Collateral.pdf

[37] http://www.euribor-ebf.eu/euribor-org/about-euribor.html

4.8 Changes in interest rate derivatives' pricing since August 2007

4.8.1 Changes in FRAs prices

4.8.1.1 Pricing short term FRAs

In Euro, 6 months Euribor Fixings are used as the standard fixing reference and Euribor futures quote the 3 month reference.

Before August 2007, 6 month Euribor two methods could be used to calculate FRAs prices:

- In a very simply manner, the rate of short term FRAs was calculated by entering into the FRA formula the value of the spot Euribor fixings. For instance the rate of a 6X12 FRA used the value of the 6 and 12 month fixings.

- Another method calculates 6 month FRA rates by using the price of two successive Euribor futures contracts since 3 and 6 month Euribor fixings were equally priced at a spread of around 6 bps against Eonia, the 6 month reference being usually quoted 1 bp. above the 3 month one.

Nowadays these simple methods of calculating FRA rates cannot be applied since 3 and 6 month Euribor fixings trade at a different level compared to Eonia. The valuation of FRAs is now based on the market spreads between Euribor swap curves and Eonia.

4.8.1.2 Hedging short term FRAs

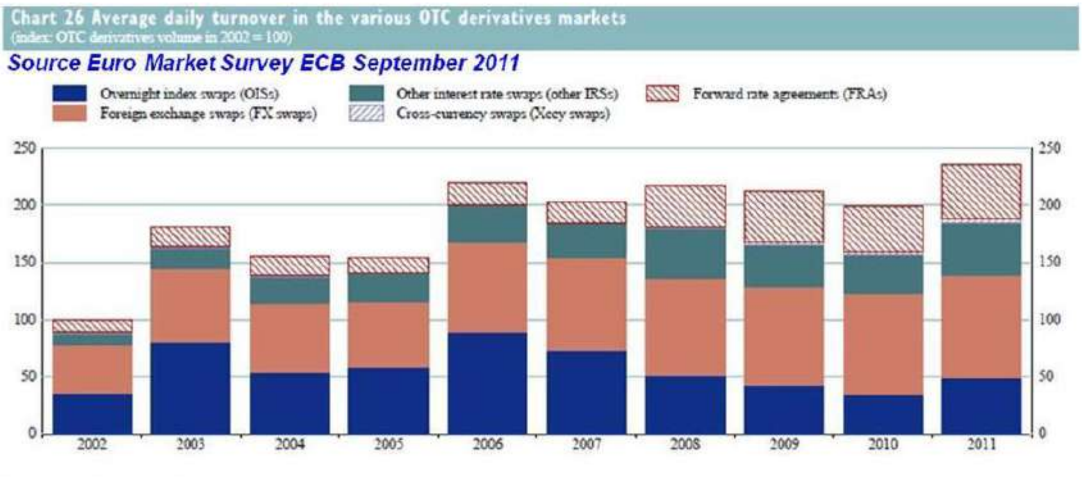
As already discussed, due to the illiquidity of unsecured money market transactions and to the stable 6-7 bps spread between Euribor and Eonia, before the onset of the financial turmoil, banks hedging a Euribor fixing contracted a same maturity Eonia swap and added to its rate a 7 bps spread.

Since then, money market conditions have totally changed:

- Same maturity Euribor Eonia swap spreads have considerably moved, having been fixed on the 6 month reference at to 2%,

- And, as indicated paragraph 4.5.1.4, this spread largely depends on the maturity of the reference suggesting that from now on, long maturity fixings incorporate an increased liquidity premium.

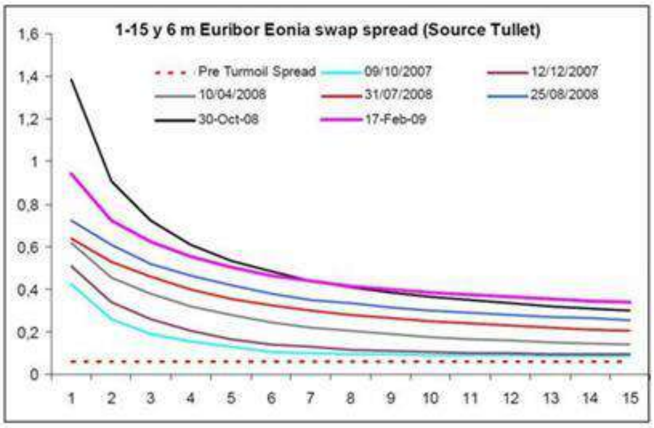
The chart following shows that, since 2007, Eonia swaps have decreased while FRSs increased, banks trying to cover their FRAs exposure with a same instrument.



Note: The panel comprised 105 credit institutions.

4.8.2 The fragmentation of the swap curve

The pricing methodology that we have discussed for FRAs also applies to the calculation of short term IRSs; and on longer maturities, swap traders have to utilize the values of the different swap curves that are now quoted in the market.



The chart opposite shows the spread differential between the 6 month Euribor curve and the Eonia one.

In spite of the regular criticisms of Euribor and Libor fixings, these references have not been replaced by a new one; in €, 6 month Euribor fixings remain the long term swap benchmark reference and Eonia swaps that are supposed to be more representative than Euribor fixings, since their rate is based on effective transactions and constitute the main reference of short term interest swaps, are by far less used, on long term derivative transactions.

For this reason, most of long term swaps referenced to a fixing different from the 6 month one are priced as basis interest rate swaps; basis swaps, instead of pricing the fixed leg rate of swaps that uses another fixing reference than the standard 6 month Euribor reference, are generally quoted at a spread between the fixed leg of swaps referenced to 6 month Euribor, on one hand, and the fixed leg applicable to this different reference, for instance the 3 month Euribor, on the other hand.

As shown on the Bloomberg TPEONIA Tullet page following, IRS swap markets now strongly differentiate swaps' interest rate according to the maturity of their reference[38].

[38] Two Curves, One Price: Pricing & Hedging Interest Rate Derivatives Decoupling Forwarding and Discounting Yield Curves, Marco Bianchetti, Banca Intesa, May 2009.

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	EURIBOR AB/6M	EONIA Act/360	EONIA EURIBOR 3M		EURIBOR Basis Swaps 1M/3M	3M/6M	6M/12M
1Y	1.588	1.139-1.189	23.00-28.00	1Y	15.50-19.50	14.20	9.20
2Y	2.019	1.551-1.601	24.50-29.50	2Y	15.70-19.70	14.10	10.70
3Y	2.354	1.882-1.932	24.60-29.60	3Y	15.00-19.00	14.00	10.50
4Y	2.615	2.139-2.189	24.70-29.70	4Y	14.30-18.30	13.80	10.00
5Y	2.829	2.358-2.408	24.40-29.40	5Y	13.80-17.80	13.50	9.45
6Y	3.004	2.537-2.587	24.10-29.10	6Y	13.30-17.30	13.10	8.95
7Y	3.145	2.686-2.736	23.50-28.50	7Y	12.90-16.90	12.65	8.45
8Y	3.259	2.811-2.861	22.80-27.80	8Y	12.55-16.55	12.15	7.95
9Y	3.356	2.918-2.968	21.90-26.90	9Y	12.20-16.20	11.65	7.50
10Y	3.444	3.020-3.070	21.10-26.10	10Y	11.90-15.90	11.15	7.10
11Y	3.528	3.114-3.164	20.30-25.30	11Y	11.65-15.65	10.70	6.75
12Y	3.602	3.201-3.251	19.50-24.50	12Y	11.40-15.40	10.25	6.45
13Y	3.667	3.276-3.326	18.70-23.70	13Y	11.15-15.15	9.87	6.20
14Y	3.720	3.338-3.388	18.10-23.10	14Y	10.90-14.90	9.48	5.98
15Y	3.758	3.385-3.435	17.50-22.50	15Y	10.65-14.65	9.10	5.80
20Y	3.827	3.489-3.539	15.50-20.50	20Y	9.00-13.00	7.65	5.20
30Y	3.659	3.362-3.412	13.20-18.20	30Y	7.10-11.10	6.00	4.50

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Chapter 2 – Pricing plain vanilla fixed income securities with the zero coupon curve

In the mid 80s, the head of an international investment bank interest rate swap derivatives desk gave a presentation to its senior management, explaining that interest rate swaps were at the cornerstone of all investment banking activities. At that time, its team employed less than 10 employees and its message was poorly received.

This chapter mainly focuses on the valuation of bonds or other fixed income instruments that bear a market price different from par, by using the swap zero coupon methodology.

It explains that, in the case of instruments bearing a price different from par, the zero coupon methodology gives a better estimation of the yield of fixed income instruments quoted in the secondary market, by comparison with the yield to maturity formula based on a flat actuarial rate.

Since the valuation of secondary transactions is performed on a very frequent basis, daily, weekly or monthly for instance, we have to interpolate the par swap yield curve yearly rates from which the zero coupon discount factors are derived, in order to match this par swap yield curve maturity with the one of the instrument that is valorized.

This chapter also describes how interest rate swap curves can measure the relative value between different bonds bearing different prices and different maturities.

1. The zero coupon methodology more accurate valuation

The CNO-FBA has published a mathematical methodology for determining Z-C rates curve. It can be found in the “Indices chapter”: “Method for calculating a CNO Zero Coupon Yield Curve”.

The use of the zero coupon methodology allows for a more precise valuation of fixed income instruments than the actuarial formula:

We recall that par swap yields do not depend on their pricing methodology, since the zero coupon curve is derived from the par swap curve.

But the yield of same price fixed income instruments, bonds or interest rate swaps, not quoted at par differs according to their actuarial or zero coupon pricing methodology,

And fixed income instruments issued by a same issuer on a same maturity can bear different yields and different prices.

In the 1st Volume[39], taking the examples of the 3.75% 25/10/2019 and of the 8.5% 25/10/2019 OATs, we have explained that two bonds issued by the same issuer on the same maturity can offer different yields to maturity, due to their difference of duration.

The same reasoning can be applied to the fixed leg of interest rate swaps, since, from a pure financial point of view, an interest rate swap can be split into two different bonds, a fixed rate one and a floating rate one.

[39] Volume A, Chapter 1 paragraph 2.2.8.2

The two Excel sheets following illustrate the difference of prices and yields between the fixed legs of two 10 year interest rate swaps, considered as 10 year fixed rate bonds. The 1st bond pays a 2% low coupon and the 2nd one, a 6% high coupon. These two bonds are priced against the same par swap curve, with a 10 rate of 3.7698%. According to the actuarial formula, the yield of these two bonds is identical, at 3.7698%, but if we price them by using the zero coupon methodology, we end up with quite diverging returns.

- The 2% bond is priced at 85.479% according to the actuarial yield to maturity formula and at 85.099% according to the zero coupon curve, showing respectively a yield of 3.7698% with the actuarial formula and 3.8208% with the zero coupon calculation.

- The 6% bond is priced at 118.298% according to the actuarial yield to maturity formula and at 118.778% according to the zero coupon curve showing respectively an actuarial yield of 3.7698% similar to the one of the 6% bond, but a yield of 3.7174%, when we use the zero coupon calculation.

	A	B	C	D	E	F
1	10 year bond paying a 2% coupon quoted below par					
2–4	Year	Par curve	Discount Factor	Zero coupon rate	Actuarial price	Zero coupon price
5					Internal rate of return	
6					3,7698%	3,8208%
7	0		1,0000		-85,479	-85,099
8	1	1,4700	0,9855	1,470	2,0000	2,0000
9	2	1,7680	0,9655	1,771	2,0000	2,0000
10	3	2,2090	0,9362	2,221	2,0000	2,0000
11	4	2,6060	0,9013	2,633	2,0000	2,0000
12	5	2,9188	0,8642	2,962	2,0000	2,0000
13	6	3,1705	0,8263	3,231	2,0000	2,0000
14	7	3,3683	0,7889	3,446	2,0000	2,0000
15	8	3,5280	0,7523	3,621	2,0000	2,0000
16	9	3,6595	0,7169	3,768	2,0000	2,0000
17	10	3,7698	0,6826	3,892	102,0000	102,0000

	A	B	C	D	E	F
1	10 year bond paying a 6% coupon quoted above par					
2–4	Year	Par curve	Discount Factor	Zero coupon rate	Actuarial price	Zero coupon price
5					Internal rate of return	
6					3,7698%	3,7174%
7	0		1,0000		-118,298	-118,778
8	1	1,4700	0,9855	4,360	6,0000	6,0000
9	2	1,7680	0,9655	4,328	6,0000	6,0000
10	3	2,2090	0,9362	4,292	6,0000	6,0000
11	4	2,6060	0,9013	4,257	6,0000	6,0000
12	5	2,9188	0,8642	4,219	6,0000	6,0000
13	6	3,1705	0,8263	4,186	6,0000	6,0000
14	7	3,3683	0,7889	4,152	6,0000	6,0000
15	8	3,5280	0,7523	4,119	6,0000	6,0000
16	9	3,6595	0,7169	4,084	6,0000	6,0000
17	10	3,7698	0,6826	0,000	106,0000	106,0000

1.2 Calculation of zero coupon rates on broken dates

The regular valuation of fixed income instruments implies the calculation of the par swap curve bearing a maturity that does not correspond to an entire number of years.

The standard day count basis is not similar on fixed income securities and on interest rate swaps: Fixed income securities use as a standard the Actual/Actual day count convention and interest rate swaps the 30/360 day count convention. However, on each of the yearly intervals of the swap curve, the valuation of fixed income instruments remains identical, whatever the day count conventions utilized. But on interest periods inferior to 1 year, the two conventions produce different numbers of days and different prices. For instance:

- With an Actual/ Actual basis the numbers of days of a quarter can range from 90 days to 92 days.

- And with the 30/360 basis a quarter is always equal to 90 days.

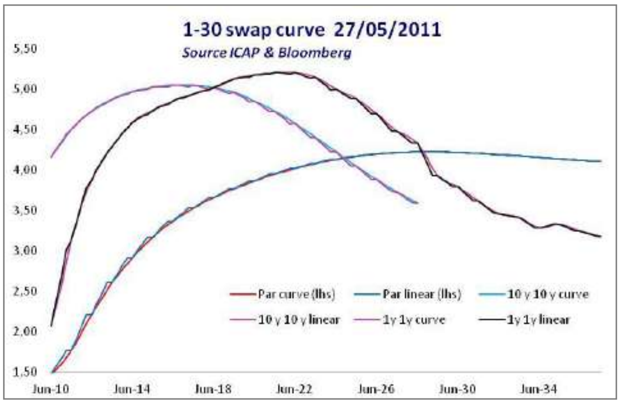
1.2.1 Linking the 30/360 par curve with the Actual/ Actual par curve

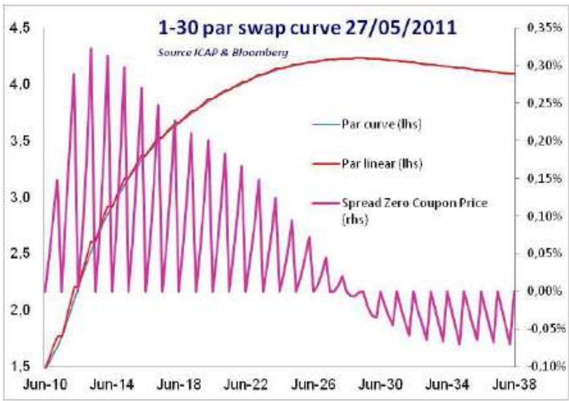
1.2.1.1 Quarterly interpolation of the 30/360 par curve[40]

The cubic interpolation defined by the CNO is based on equal interpolated periods of time, for instance yearly periods of 360 days or quarterly periods of 90 days, as it is the case with the standard IRS 30/360 day count basis. According to this market convention, each quarterly period of interest is equal to 90 days.

- The 1st chart following illustrates the quarterly interpolation of the 1-30 par swap curve on 27 May 2011 using the CNO cubic interpolation, on one hand, and a linear interpolation, on the other hand. It shows the regular shape of the 1 year forward and 10 year forward swaps derived from the par curve.

- However the price in percentage of a zero coupon bond that is identical on yearly coupon dates can strongly differs on broken dates, depending on the interpolation calculation used between two yearly par swaps rates. The right hand scale of the 2nd chart shows the price difference of 1-30 year quarterly zero coupon bonds, priced either with a cubic interpolation or a linear one.





Not surprisingly, the benefit of a cubic interpolation instead of a linear one increases with the steepness of the yield curve. The 27/05/2011 swap curve that we have used in the above example is relatively steep, since its 10-2 swap spread amounts to 120 bps. Since July 2005, this spread has ranged from –60 bps to 200 bps, with an average of 85 bps during this period. The following table gives some historical examples of the changes in price of a zero coupon consecutive to a change in the interpolation methodology.

Historical change in zero coupon prices between a cubic and a linear interpolation of the par swap curve

Date	1-juil.-08	13-sept.-05	14-sept.-11	27-mai-11	01/06/2009
10-2 y spread	-0,63	0,85	1,06	1,20	2,00
Max change in price	-0,06%	0,17%	0,20%	0,20%	0,32%

40 The interpolation methodology described in this document with a quarterly interpolation of the 30/360 swap curve can also be applied to a monthly interpolation based on a theoretical curve applied to a 360 day curve made of 12 months bearing 30 days.

1.2.1.2 Mixing 30/360 and Actual/Actual basis on the swap last interest period

The interpolation methodology of the par swap curve applied to the valuation of a cash flow bearing a specific broken date is applied in the following manner:

- The last interest period is considered as the broken date period, all the preceding periods bearing the yearly periodicity of the euro par swap curve[41]. Its number of days is calculated, using the Actual/Actual day count basis, between this broken date and the maturity date of the immediate preceding yearly Discount Factor.

- The yield to maturity of the par swap curve applied to this specific broken date is obtained by linearly interpolating the values of the two par swap quarterly rates (calculated by cubic interpolation) that surround this specific broken date. Since the maximum distance between at least one of these two quarterly rates and the specific broken date is equal to 45 days, this linear interpolation can be considered as acceptable.

- Calculation of the Discount Factor applicable at maturity of the *n* broken date: We calculate the value of the principal amount that together with the interest paid at maturity of the last broken period will produce a repayment of € 1 and then deduct from this principal amount the present value of the preceding yearly coupons (calculated forward since the settlement date) paid on this principal amount according to the following formula:

$$DF_n = \left(\frac{1 - PSR_n * \sum_{DF_{a1}}^{DF_{an-1}}}{(1+PSR_n)^{(date_n - date_{n-1})/ActAct_{Basis}}} \right) \qquad (B.1.3.5)$$

---

erreur nombre de jour 31/5/21 – 31/2/22 ne fait pas 365

[41] The choice of a last broken period of interest instead of a 1st broken period seem surprising compared with the method that we recommend in the 1st volume Chapter 1 Paragraph 3, to price fixed income securities. However, it is largely justified:

- Since the zero coupon yearly curve is calculated step by step starting from the 1st year, each of these yearly rates is based on the most robust points of both the par curve and the zero coupon curve. Setting the broken period as the 1st interest period would mean that all the other yearly zero coupon rates would depend on the interpolation methodology employed to calculate this first coupon, potentially creating some noise. To the contrary, in the actuarial world used to price fixed income securities, this consideration is irrelevant since all coupons are compounded at a unique flat rate.
- This choice is also more suitable for the valuation of a portfolio of several swaps or fixed income securities, since in this case, apart from their last interest rate period, all these instruments are priced against the same preceding yearly discount factors.
- According to the CNO methodology described for bonds on the 1st Volume, the life to maturity of a fixed rate instrument bearing a 1st short coupon is identical to the life to maturity of a fixed rate instrument bearing a last short coupon[41].
-

The table following illustrates the calculation of the par swap rate and Discount Factor applicable on a broken date according to the above methodology.

Act/Act par swap curve interpolation on a specific broken date

	Dates	Par Swap Rates*	Discount Factor	Day count
Settlement date	31-mai-11			
30/360 Next quarterly curve interpolated date	27-mai-22	3,3968000	0,68647878	
30/360 Specific interpolation broken date :	**15-avr.-22**	**3,3822208**	**0,6906696**	
30/360 Previous quarterly curve interpolated date	27-févr.-22	3,3775195	0,6939390	
30/360 Preceding yearly zero coupon date	31-mai-21		0,7166318	
Actual number of days of the last atypical coupon between 31/052021 & 15/04/2022				319
Actual number of days between 31/05/2021 & 31/02/2022				365

** Rates are expressed as yearly rates compounded annualy*

1.2.1.3 Usual market conventions applicable to primary swaps bearing a broken maturity

The above description of the calculation of the Discount Factor applicable on a broken date that sets the broken date as the last interest period corresponds to the pricing of a swap or a swap portfolio in the secondary market based on a par swap curve paying yearly coupons.

This valuation methodology also applies to the rate of a new, primary, swap transacted on a broken period. However, the contract applying to a primary swap transacted on a broken date will set the broken period as its 1st coupon period, not as the last one:

- Setting the broken period as the 1st period of interest or the last one does not change the value of the swap and, as in the bond market, it is simpler to enter into the systems, since after the 1st coupon payment, all subsequent coupons paid on the swap fixed leg bear a yearly periodicity,

- And the rate applicable on this 1st broken period is not decompounded. The facial swap rate applicable to the 1st broken coupon period is identical to the one applicable to its subsequent yearly coupons. Therefore, it is expressed as the simple rate that will be applied to the nominal amount of the swap transaction in proportion of the number of days of the coupons' periods. This difference of methodology affects the yield to maturity of the transaction, especially with a high level of interest rate and a short maturity.

2. Mark to market of secondary swaps

By definition, the present value of a primary par interest rate swap is equal to zero, the value of its fixed leg being equal to the value of its floating leg. Once transacted, this swap is qualified as a secondary swap and its historical price depends on the change in rates applicable to both its contractual fixed leg and the fixed reference of its current floating coupon.

Mark to Market of a 100 M swap maturing on 4/10/2013

			+ Fixed leg		- Floating leg	
Valuation date		01/06/09	+ Fixed leg		- Floating leg	
Fixed coupon starts		04/10/08	Gross NPV price		Gross NPV price	
Floating coupon starts		04/04/09	110,22%		-102,538%	
Net NPV total price 5,073%			Net NPV price 107,27%		Net NPV price -102,19%	
	Spot rates	Disc factors	Accrued interest 2,96		Accrued interest -0,3457	
01/06/09		1,000	Flows	NPV	Flows	NPV
01/09/09	1,266	0,9968				
04/10/09	1,338	0,9954	4,50	4,48	-101,0908*	-100,62
01/12/09	1,464	0,9926				
04/01/10	1,465	0,9912				
01/03/10	1,467	0,9890				
04/04/10	1,468	0,9876			-0,253	-0,250
01/06/10	**1,470**	**0,9855**				
04/07/10	1,492	0,9840				
01/09/10	1,531	0,9812				
04/10/10	1,556	0,9795	4,50	4,41	-0,254	-0,249
01/12/10	1,601	0,9764				
04/01/11	1,631	0,9745				
01/03/11	1,680	0,9712				
04/04/11	1,713	0,9692			-0,253	-0,245
01/06/11	**1,768**	**0,9655**				
04/07/11	1,804	0,9632				
01/09/11	1,869	0,9591				
04/10/11	1,910	0,9565	4,50	4,30	-0,254	-0,243
01/12/11	1,982	0,9519				
04/01/12	2,026	0,9491				
01/03/12	2,098	0,9442				
04/04/12	2,139	0,9413			-0,254	-0,239
01/06/12	**2,209**	**0,9362**				
04/07/12	2,246	0,9333				
01/09/12	2,313	0,9279				
04/10/12	2,350	0,9248	4,50	4,16	-0,254	-0,235
01/12/12	2,416	0,9192				
04/01/13	2,453	0,9159				
01/03/13	2,514	0,9103				
04/04/13	2,548	0,9070			-0,253	-0,229
01/06/13	**2,606**	**0,9013**				
04/07/13	2,637	0,8980				
01/09/13	2,692	0,8921				
04/10/13	**2,721**	0,8887	**104,50**	92,87	**-0,254**	**-0,224**
01/12/13	2,771	0,8829				
04/01/14	2,847	0,8774				
01/03/14	2,919	0,8736				
04/04/14	2,987	0,8651				
01/06/14	**3,052**	**0,8642**				

* 6 m Euribor rate (1.646% + 0.50%)*183/360 + Principal 100

The valuation of a secondary swap simulates the conclusion of a reversal swap, a swap that hedges the market rate exposition originally generated by this secondary swap.

2.1 simulation of a reversal swap

This reversal swap bears the same maturity date and the same floating reference as the ones of the swap that is valorized. In order to match the fixing dates and coupon periods of these two swaps, the reversal swap bears a short 1st coupon.

In the case of a secondary fixed rate receiver swap, this valuation will simulate a primary fixed rate payer swap[42].

We take the example of a 5 year interest swap contracted on 4 October 2008, where a corporate receives a yearly fixed rate of 4.5% and pays a floating reference of 6 month Euribor + 0.50%. The table opposite illustrates the valuation of the fixed and floating legs of this swap using the zero coupon curve, on 1 June 2009.

2.2 Valuation of the swap fixed leg

This reversal swap prices, at the calculation date market conditions, the net present value of the primary swap on its residual maturity after having reintroduced in its cash flows the repayment at maturity of its principal amount. As in the case of a bond, the gross price of this fixed leg incorporates the value of the fixed rate coupon that has been accrued between the beginning of the fixed coupon period and the settlement date applicable at the calculation date.

2.3 Valuation of the swap floating leg

We valorize treat differently the 50 bps fixed margin paid on the swap 1st coupon on one hand, and the same 50 bps margin paid on its subsequent not yet fixed Euribor coupons, on the other hand:

- We add this 50 bps margin to the current already fixed 6 month Euribor fixing and valorize this current coupon as a fixed rate bond by discounting it against the money market offered rate that applies to its remaining maturity.

- Then we exclude this fixed margin from the subsequent not yet fixed floating rate coupons up to the swap maturity, since as later explained we will treat this margin separately as an annuity.

2.3.1 We price the fixed 1st coupon as a fixed bond maturing at the end of this coupon period

- The floating coupon is treated as a fixed interest rate since its fixing has already been determined. The reversal swap valorizes the change in rates applicable to the already fixed 1st floating coupon, including the 50 bps margin paid in our example at the end of the current coupon period.

- Then we valorize the subsequent floating coupons without adding the 50 bps margin to the fixing. By doing so, the Euribor subsequent coupons paid on the secondary swap and received on the reversal swap are identical and can be neutralized.

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[42] Vice versa, in the case of a secondary payer swap, the reversal swap will simulate a primary receiver swap.

- Consequently, we consider that, at each fixing date, the present value of both secondary and reversal swaps floating legs is equal to par. For this reason, by simplification, in the valuation of the secondary swap that excludes its 50 bps, we can consider that its floating leg maturity is equal to the maturity of its already fixed 1st floating coupon payment date.

2.3.2 The fixed margin paid on top of the Euribor is considered as a fixed rate

However since, in our chosen example, the floating leg of the secondary swap pays a margin of 50 bps on top of the Euribor floating reference, we consider this 50 bps margin as a fixed rate annuity paid every 6 months and we valorize these semiannual payments against the zero coupon curve.

2.3.3 Comparison between the results of the zero coupon and actuarial methods

The fixed par swap rate of the reversal swap applicable to the secondary swap maturity is equal 2.721% and the floating rate applicable to the maturity of the current floating coupon is equal to 1.338%.

This interest swap bears a relatively short maturity, and the gross price of its fixed leg calculated with the zero coupon rates, at 110.22%, is not that far from the price of 110.14% that we obtain by discounting these cash flows at an actuarial flat rate of 2.721%. However expressed in rate, this difference of price is equal to 2 bps, which in the case of IRSs is not acceptable.

3. Pricing plain vanilla fixed income securities against the swap curve

This valuation can respond to two main purposes, a theoretical valuation exercise, in order to select the bond offering the best fixed return, as exposed on paragraph 5.1, or the acquisition of asset swap packages, in order to secure a fixed margin against a floating rate exposure and to reduce its investor interest rate risk, as exposed on paragraph 5.2.

3.1 Theoretical valuation of bonds' yield relative value

The comparative valuation of fixed income instruments against an interest swap curve can be performed according to two different methodologies, the flat yield to maturity actuarial formula or the derivative zero coupon methodology:

3.1.1 The approximate actuarial yield to maturity asset swap spread

According to its definition, the relative value of a fixed rate bond is calculated by simply deducting from its yield to maturity the rate of a same maturity par swap, irrespective of the bond market price. For instance, the yield to maturity of a 2% 10 year bond discounted at a flat yield of 4.070%, corresponds to an actuarial price of 83.270%, as illustrated by the Excel file following.

- In this example, the actuarial asset swap margin of this bond is equal to 30 bps, when compared to the 10 year par swap rate of 3.770%.

- However, since this bond is not priced at par, this yield to maturity formula does not reflect the bond credit risk. In the case of a steep yield curve, if the bond market price trades below par, the yield to maturity calculation underestimates the bond credit risk and if it trades above par, the yield to maturity calculation overestimate its credit risk. On a same maturity, a bond priced below par has a longer duration than a par bond; and a bond priced above par has a shorter duration than a par bond.

- As shown on this Excel file, the same 10 year 2% bond valorized according the zero coupon methodology is priced at 82.837% (compared to 83.270 with the yield to maturity method) corresponding to an actuarial yield to maturity of 4.130%, i.e. at a 36 bps spread above the par swap curve.

- Consequently, according to these two methodologies, this same bond asset swap margin differs by 6 bps.

In this example the zero coupon rate is derived from the par swap rate to which has been added the 30 bps swap margin.

10 year bond paying a 2% coupon quoted below par

Year	Par curve flat	Par curve + 30 bps	Discount Factor	Zero coupon rate par curve + 30 bps	Actuarial price	Zero coupon price
					Internal rate of return	
					4,070%	4,130%
0			1,0000		-83,270	-82,837
1	1,470	1,7700	0,9826	1,7700	2,0000	2,0000
2	1,768	2,0680	0,9598	2,0711	2,0000	2,0000
3	2,209	2,5090	0,9280	2,5228	2,0000	2,0000
4	2,606	2,9060	0,8907	2,9359	2,0000	2,0000
5	2,919	3,2188	0,8515	3,2667	2,0000	2,0000
6	3,171	3,4705	0,8117	3,5372	2,0000	2,0000
7	3,368	3,6683	0,7727	3,7530	2,0000	2,0000
8	3,528	3,8280	0,7347	3,9297	2,0000	2,0000
9	3,660	3,9595	0,6979	4,0773	2,0000	2,0000
10	3,770	4,0698	0,6625	4,2029	102,0000	102,0000

3.1.2 The Z spread

The Z spread refers to the Zero Coupon methodology. It expresses the number of bps that needs to be added to the zero coupon yield curve derived from the par swap curve, so as to equalize a specific bond market price with the discounted value of its future cash flows.

Contrary to the calculation made in the previous paragraph, the Z spread is directly applied to the zero coupon curve derived from the par swap curve.

The Excel file following compares the yield of a 2% 10 year bond according to the actuarial and the Z spread methodology:

- The 83.270% market price of this bond corresponds to an actuarial yield of 4,070%, the 10 year swap rate plus a 30bps margin.

- But when we price it against the zero coupon curve, in order to obtain the same 83.270% price as the one calculated with the flat yield of 4.070%, we end up with a Z spread of 24.99 bps.

This 5 bps difference is due to the longer duration of this low coupon bond compared to that of the par swap.

2% 10 y bond priced at a flat actuarial yeld of 30 bps over 10 y swap

Year	Par curve flat	Discount Factor	Zero coupon rate	Zero coupon spread bps	Actuarial price	Zero coupon price
				Z spread	Internal rate of return	
				24.99	4.070%	4.070%
0		1,0000			-83,270	-83,270
1	1,470	0,9855	1,4700	0,2499	2,00	2,00
2	1,768	0,9655	1,7706	0,2499	2,00	2,00
3	2,209	0,9362	2,2211	0,2499	2,00	2,00
4	2,606	0,9013	2,6328	0,2499	2,00	2,00
5	2,919	0,8642	2,9621	0,2499	2,00	2,00
6	3,171	0,8263	3,2313	0,2499	2,00	2,00
7	3,368	0,7889	3,4458	0,2499	2,00	2,00
8	3,528	0,7523	3,6213	0,2499	2,00	2,00
9	3,660	0,7169	3,7678	0,2499	2,00	2,00
10	3.770	0,6826	3,8924	0,2499	102,00	102,00

In this 10 year bond example the spread difference between the actuarial yield to maturity asset swap margin and the Z spread amount to 1 bps; this can be considered as quite negligible by people not familiar with the derivative market. However, 1 bp over a 10 year period represents a great value for swap traders.

And in the case of bonds traded in the market at a much lower price and on longer maturities, the difference of spread between the two methodologies can be sensibly larger than 1 bp.

3.2 Implementing asset swap packages

Asset swaps package are mainly used by fixed income investors seeking to reduce their interest rate exposure and to secure a fixed margin over their own cost of funding, on a Euribor basis.

3.2.1 Asset swap definition

We take the example of the 2% 10 year bond priced in the market at 83.270%, as already illustrated in the previous paragraph, to describe a par asset swap package, a structure mainly used in the asset swap market.

A par asset swap combines two different market transactions:

- The bank sells to its asset swap investor this 10 year 2% bond at a par price, 100%, whatever its market price; at the same time, the bank contracts with this investor an interest rate swap, on the bond maturity for a notional amount equal to the nominal amount of the bond: For instance on a 10 million asset swap transaction, the investor pays the bond 2% coupon and receives from the bank 6 month Euribor plus a margin, on this notional amount of 10 million.

- The bank purchases in the market the 2% 10 year bond at its current price, in our example at 83.270%, making a gain of 16.73% between the market price of the bond and the par price paid by the investor.

3.2.2 Asset swap cash flows

The chart below details the cash flows of this 10 year par asset swap against 6 month Euribor:

- The investor pays at the transaction date, the 16.73% upfront payment between its par purchase of the bond and the price that the bank has paid in the market and, on a yearly basis, the 2% coupons that it receives over the maturity of the transaction; in exchange it receives 6 month Euribor plus a fixed rate margin. As long as this investor is able to borrow 6 month deposits at Euribor flat, it will earn the bond asset swap margin.

- On the fixed leg, the bank counterparty receives, at the asset swap inception the 16.73% upfront payment and, on a yearly basis, the bond 2% coupons. On the floating leg, it pays to its investor 6 month Euribor plus a margin that corresponds to the spread between the yield of the bond and the 10 year par swap rate, on one hand, and to the annualized value of the 16.73% upfront payment, on the other hand.

- The bank covers its interest rate exposure in the interbank swap market, paying a 3.77% fixed rate and receiving 6 month Euribor flat.

3.2.3 The influence of the bank counterparty cost of funding on the asset swap spread

Since August 2007, the long term cost of funding of many international banks has largely increased and, in some cases, these banks have to pay a higher spread over the Euribor floating reference than the one paid by the bond that is asset swapped.

This situation deteriorates the asset swap margin of a par asset swap package when the bank has to pay in the market the bond well above par; in such a case, the bank has to finance as an annuity, up to the bond maturity, the price differential between the bond market price and par.

The Excel sheet following illustrates the par asset swap pricing of a theoretical bond paying an 8% coupon on a 25 year maturity. This bond is bought by the bank at its gross market price of 138.27% and sold back to the investor at par against 6 month Euribor + 50 bps. Consequently the bank has to finance the 38.27% that correspond to the future value of the rate differential that the bank will receive on a yearly basis between the 8% coupon and the Euribor 25 year swap + 50 bps.

- This calculation has been made by assuming that the bank counterparty will refinance at Euribor + 50 bps the 38.27% that it has to refinance over the bond maturity.

- But if the cost of funding of the bank counterparty amounts to 100 bps, 50 bps above the bond asset swap margin, the bank has to decrease by 10 bps, at 40.2 bps the spread it can offer on this asset swap package.

	A	B	D	F	G	H	J	K	L
1		Influence of the bank counterparty funding cost on a par asset swap							
2		Bond maturity	25-juil-32	Gross market price in %		138,27	Net price in %		138,270
3		Coupon	8	Yield		5,224	Accrued interest		0
4		Settlement date	25/07/2007						
5		Bond market price at Euribor +			50	138,27	Price differential		38,27
6									
7				The bank receives			The bank pays Euribor +		50 bps
8							Cash flows'		
9	Year			Nominal		NPV	differential	NPV	NPV
10	0	25-juil.-07	Coupons	Curve	Disc Fac	145,743	Current	38,27	7,475
11	1	25-juil.-08	8,000	4,578	0,956	7,650	2,66	2,54	0,497
12	2	25-juil.-09	8,000	4,633	0,913	7,307	2,66	2,43	0,475
13	3	25-juil.-10	8,000	4,644	0,873	6,981	2,66	2,32	0,453
14	4	25-juil.-11	8,000	4,652	0,834	6,669	2,66	2,22	0,433
15	5	25-juil.-12	8,000	4,656	0,796	6,372	2,66	2,12	0,414
16	6	25-juil.-13	8,000	4,667	0,760	6,084	2,66	2,02	0,395
17	7	25-juil.-14	8,000	4,677	0,726	5,808	2,66	1,93	0,377
18	8	25-juil.-15	8,000	4,691	0,693	5,541	2,66	1,84	0,360
19	9	25-juil.-16	8,000	4,707	0,661	5,284	2,66	1,76	0,343
20	10	25-juil.-17	8,000	4,726	0,629	5,036	2,66	1,67	0,327
21	11	25-juil.-18	8,000	4,741	0,600	4,799	2,66	1,60	0,312
22	12	25-juil.-19	8,000	4,761	0,571	4,568	2,66	1,52	0,297
23	13	25-juil.-20	8,000	4,776	0,544	4,349	2,66	1,45	0,282
24	14	25-juil.-21	8,000	4,790	0,517	4,140	2,66	1,38	0,269
25	15	25-juil.-22	8,000	4,807	0,492	3,937	2,66	1,31	0,256
26	16	25-juil.-23	8,000	4,812	0,469	3,752	2,66	1,25	0,244
27	17	25-juil.-24	8,000	4,819	0,447	3,574	2,66	1,19	0,232
28	18	25-juil.-25	8,000	4,824	0,426	3,405	2,66	1,13	0,221
29	19	25-juil.-26	8,000	4,827	0,406	3,246	2,66	1,08	0,211
30	20	25-juil.-27	8,000	4,828	0,387	3,095	2,66	1,03	0,201
31	21	25-juil.-28	8,000	4,829	0,369	2,951	2,66	0,98	0,192
32	22	25-juil.-29	8,000	4,829	0,352	2,816	2,66	0,94	0,183
33	23	25-juil.-30	8,000	4,827	0,336	2,688	2,66	0,89	0,175
34	24	25-juil.-31	8,000	4,824	0,321	2,567	2,66	0,85	0,167
35	25	25-juil.-32	108,000	4,8199	0,307	33,124	2,66	0,82	0,159

- Although the standard I/L swap curve is priced against 6 month Euribor, the Euribor asset swap market is generally transacted against 3 month Euribor, since asset swap investors have an easier access to 3 month funding than to the 6 month one.

- However 6 month Euribor asset swaps are also quoted in the market and this Excel file price this 25 year 8% bond asset swap against the 6 month curve, in order to reduce the size of the Excel files that we use to illustrate the different pricing methodologies of I/L bond asset swaps; the par notional and the accreting notional one divergence of valuation increases on long maturities.

- This Excel file publishes only the yearly payments on both fixed and floating leg of the swap. Consequently, we consider that the 6 month Euribor + 50 bps coupons are paid on a yearly basis being compounded twice a year and on a yearly basis, the 50 bps Euribor spread paid semiannually corresponds to a 52 bps paid once a year.

Chapter 3 – IRS paying a long term CMS or CMT floating reference

This chapter refers to the 3rd chapter of the 1st Volume of this document describing international fixed income instruments referenced to a long term Constant Maturity Rate, like the Constant Maturity Swap fixing used in the international markets published by ISDAFIX, the US Constant Maturity Treasuries rate published by the FED, the French Taux de l'Echéance Constante, CNO-TEC10®, published by the French Bond Association, or the 15 year floating rate JGBs[43].

A Constant Maturity Reference is the yield of fixed rate instrument bearing a constant maturity. This reference can either refer to:

- A Constant Maturity Swap rate, CMS, corresponding to the daily fixings of the par swap curve published daily on the Reuters ISDAF2 page,
- Or the yield of a notional Constant Maturity Treasury, calculated by interpolating a Treasury Yield Curve such as the US CMT or the French CMT, the latter reference being known as the CNO-TECn®.

1. The utilization of CMT or CMS references

Constant Maturity rates are used in the fixed income swaps as the long term floating reference of:

- Either plain vanilla swaps or bonds, like the EIB TEC 10 25 Jan 2020 bond described in the 1st Volume of this document. These floating rate instruments are largely sought after by long term investors, pension funds or French insurance companies, exposed to a rise in long term interest rates.
- Or exotics structures like private placements with coupon indexed on a spread of CMS, these structures being mainly sold to retail investors willing to increase their current return: A large European investment bank issued in July 2007 a private placement on a 15 year maturity paying:
    - ✓ A fixed rate of 10% during the 1st two years,
    - ✓ And a coupon equal to 50 times the CMS 30 – CMS 10 spread afterwards.

This chapter concentrates on the derivative valuation of plain vanilla CMS or CMT instruments that pay the reference of a sole long term interest rate. It will not develop the valuation of the exotic private placements based on long term swap spreads. These instruments do not generally correspond to the CNO definition of fixed income securities described in the 1st Volume of this document.

In the US and in France, CMT or CMS references are also largely used for various other purposes, like the indexation of floating rate mortgages or car leases for example; so as to hedge these plain vanilla CMS or CMT references, a market of Constant Maturity Swap has developed:

- Some interbank brokers quote on their screens the rate of 5, 10, 15 and 20 year CMS swaps against 3 month Euribor. However, CMS swaps are barely traded in the interbank market,

[43] The Japanese Ministry of Finance is by far the largest issuer of CMT CMT bonds having issued between 2002 and 2008 the equivalent of € 380 billion of 15 year floating rate JGBs referenced to the yield of their last 10 year auction.

investment banks hedging a CMS position by replicating this position with various plain vanilla interest rate swaps.

- Other positions on CMT or TEC10 swaps are also hedged on the CMS market and in this case, the bank has additionally to cover its CMS-CMT exposition with an appropriate basis swap named spread lock.

2 CMS swaps' definition and general characteristics

Constant Maturity Swaps, CMS, exchange two floating rates:

- A standard Euribor leg generally indexed to the 3 month fixing,

- And a CMS leg that pays every 3 months a long term swap rate, for instance the 10 year swap fixing against the 6 month Euribor fixing published daily at 11:00 CET on the Reuters ISDAF2 page, from the contribution of the 15 panel banks that participate to this fixing, as shown on the opposite table[44].

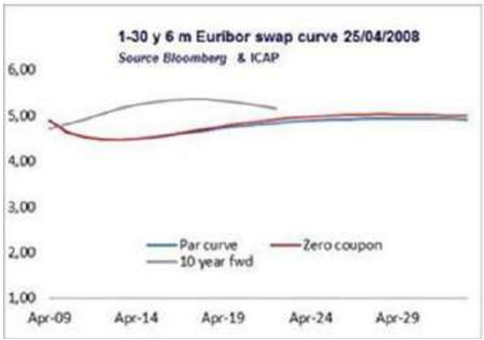
ISDA FIX CHARACTERISTICS AND CONTRIBUTOR PANELS

EUR Euribor

Maturities	Contributors
1-10, 12, 15, 20, 25, 30	Bank of America Merrill Lynch
	Barclays Bank
	BNP Paribas
Day Count (rates)	Citibank
Annual 30/360 vs. 6 month Euribor	Commerz
For the 1 year maturity: Annual	Credit Suisse
30/360 vs. 3 month Euribor	Deutsche Bank
	Goldman Sachs
Business Days: TARGET	Hypo Vereinsbank
	JP Morgan Chase
Business Day Convention:	Morgan Stanley
Modified Following	Rabobank
	Royal Bank of Scotland
	Société Générale
Reuters page	UBS
ISDAFIX2	*Updated June 2010*

2.1 Specific characteristics of interbank CMS conventions

Interbank CMS quoted on interdealer broker screens bear an uncommon mélange of market conventions. The left chart below illustrates the Fixed rate/ 6 month Euribor par swap curve on 25/05/2008 and the ICAP Reuters page on the right publishes the quoted margin of 5 to 30 year CMS swaps bearing a CMS2, CMS5 or CMS10 index. Since the par swap curve is rather flat, the value of the CMS quoted margins is relatively small, a CMS10 at 15 year paying a 50.9 to 52.9 bps quoted margin. We will illustrate the conventions of these CMS by using the example of a 15 year CMS paying a CMS10 index:



12:56 25APR08 ICAP Constant Maturity Swaps

		2Y Index	5Y Index	10Y Index
5Y	Swaps	9.7-10.9	24.0-25.5	55.2-57.2
10Y	Swaps	19.6-20.8	36.0-37.5	60.4-62.4
15Y	Swaps	20.6-21.8	33.2-35.1	50.9-52.9
20Y	Swaps	20.3-21.8	30.1-32.4	43.4-45.9
30Y	Swaps	20.2-22.2	28.2-31.4	40.1-45.1

Prices Quoted Q A/360 vs 3M Euribor

- The quarterly fixings of the 10 year reference are published on the ISDAFIX Reuters page, 2 working days before the swap settlement. These fixings correspond to the 10 year par swap fixed rate paid on a yearly periodicity with a 30/360 day count fraction. And this rate use the 10 year fixed rate/ 6 month Euribor swap curve.

[44] http://www.isda.org/fix/pdf/ISDAFIXEUR-Euribor.pdf

- Every 3 month the CMS leg of a 15 year CMS/ Euribor swap pays the 10 year fixing corresponding to the yearly coupons of the reference without decompounding this yearly rate in a quarterly rate. For instance if the 10 year swap rate is fixed at 4.5672% this rate will not be transformed into a quarterly equivalent of a yearly rate.

- However CMS coupons paid quarterly are as indicated on the ICAP page applied on an Actual/360 day count fraction. Therefore the 4.5672% yearly rate of a swap using a 30/360 basis will be converted into an Actual/360 basis of 4.5046%.

- And the quarterly coupons paid by the CMS swap incorporate the 3/ 6 month Euribor basis applying on the CMS swap maturity.

2.2 Valuation of constant maturity swaps against the plain vanilla par swap curve

We first concentrate on the valuation of pure CMS swaps, interest rate swaps bearing a specific Constant Maturity Swap reference by opposition to:

- Swaps referenced to other constant maturity references like a sovereign bond index, the US CMT rates or the French TEC 10 rates, on one hand,

- And to bonds referenced to any Constant Maturity reference, on the other hand.

Pure CMS swaps' price sensitivity to the changes in interest rate conditions is different to the one of plain vanilla fixed rate instruments.

The specific curve price sensitivity of CMSs cannot be expressed by the modify duration concept; we will expose in the following paragraph how we can measure this specific curve price sensitivity.

Constant Maturity swaps and bonds based on other Constant Maturities references than the pure CMS one can be analyzed as a combination of a pure CMS position, on one hand, and a fixed rate spread that can be priced with the standard modify duration formula, on the other hand. In the case of a TEC 10 reference, this spread, named spread lock, represents the anticipated swap spread between 10 year OATs and 10 year swaps, during the life of this Constant Maturity Swap.

2.2.1 The specific valuation of CMS swaps

The Constant Maturity coupons of a 15 year CMS swap paying every 3 month a 10 year par swap reference are priced as 10 year forward rates: The first coupon uses the 10 year spot rate. All subsequent coupons are priced as 10 year par forward rates, from the 10 year in 3 months, to the 10 year in 14 years and 9 months. These par forward rates are calculated using the zero coupon discount factors that are derived from the par swap curve, as explained in the 1st chapter of this document. The Excel sheet opposite illustrates the calculation of the 10 year par forward rates in 10 years using the zero coupon Discount Factors from the year 5 to the year 15.

6 m Euribor par and zero coupon swap curve, zero coupon & Forward curve

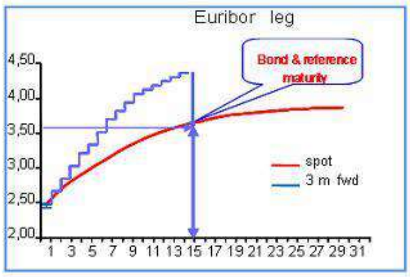
Year	01/06/2009	Par curve	Discount Factor	Zero coupon	1 year fwd	5 year Fwd	10 year fwd
1	1-juin-10	1,4700	0,9855	1,470	2,07	3,543	4,160
2	1-juin-11	1,7680	0,9655	1,771	3,13	4,092	4,501
3	1-juin-12	2,2090	0,9362	2,221	3,88	4,449	4,732
4	1-juin-13	2,6060	0,9013	2,633	4,29	4,670	4,879
5	1-juin-14	2,9188	0,8642	2,962	4,59	4,821	=(G17-G27)/(SOMME(G18:G27))*100
6	1-juin-15	3,1705	0,8263	3,231	4,74	4,931	5,029
7	1-juin-16	3,3683	0,7889	3,446	4,86	5,019	5,055
8	1-juin-17	3,5280	0,7523	3,621	4,95	5,091	5,053
9	1-juin-18	3,6595	0,7169	3,768	5,02	5,145	5,021
10	1-juin-19	3,7698	0,6826	3,892	5,13	5,176	4,967
11	1-juin-20	3,8673	0,6493	4,004	5,18	5,156	4,862
12	1-juin-21	3,9508	0,6173	4,102	5,21	5,102	4,734
13	1-juin-22	4,0228	0,5867	4,187	5,21	5,004	4,580
14	1-juin-23	4,0838	0,5577	4,259	5,15	4,863	4,408
15	1-juin-24	4,1338	0,5304	4,319	5,01	4,700	4,230
16	1-juin-25	4,1710	0,5051	4,362	4,89	4,490	4,053
17	1-juin-26	4,1990	0,4815	4,393	4,70	4,271	3,888
18	1-juin-27	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

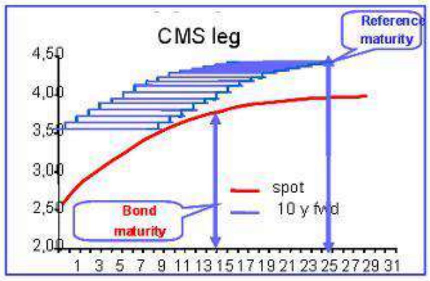
2.2.2 CMS swaps priced against the Fixed/ Euribor par swap curve

CMS/ Euribor and Fixed rate/ Euribor swaps are priced against the par swap curves published by interdealer brokers and their two legs should offer the same yield, since interest rate swaps are priced by arbitrage:

- In the case of a new Fixed/ 3 m Euribor swap priced at par, the yield of its Euribor leg forward coupons is equal by construction to its fixed leg yield. The left chart below illustrates the forward values of the floating leg of a 15 year Fixed/ Euribor swap that are calculated by using the zero coupon Discount Factors, from the 3 month in 3 months up to the 3 month in 14 years and 9 months.

- The CMS forward coupons of a par CMS/ Euribor paying quarterly a 10 year reference are also calculated by using the value of the 10 year par forward swaps from the 10 year in 3 months up to the 10 year reference in 14 years and 9 months, as shown on the right chart below.





- But, with a steep yield curve, the yield of these forward CMS 10 year floating references is higher than the yield of its forward Euribor leg, since the maturity of the CMS reference extends beyond that of the CMS bond, as shown on the right chart above. A 15 year CMS bond paying quarterly a 10 year reference is priced using all the quarterly steps of the swap curve from 3 months up to 25 years. Since the two legs of a swap priced at par have to be equal, we will later on explain that the par CMS swap curve has to incorporate a fixed margin named "quoted margin" that equalizes the value of its CMS leg with that of its Euribor leg.

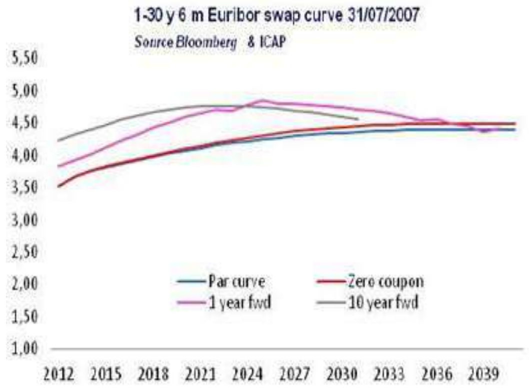
2.3 Comparison of Fixed/ Euribor and CMS/ Euribor curve price sensitivity

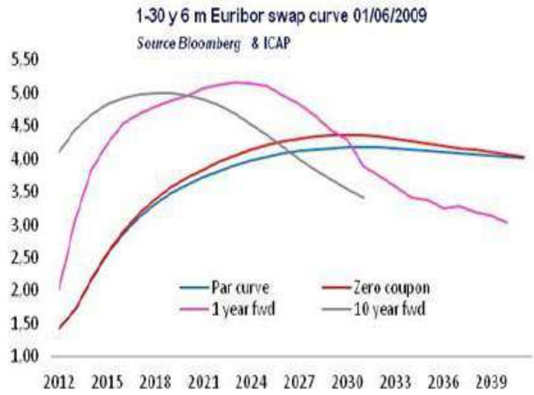
- A parallel shift of the par swap curve will affect the price of a Fixed/Euribor swap, but a change in the slope of the par yield curve has nearly no influence on the value of this swap, as long as the fixed rate corresponding to its maturity does not move, since the last floating rate Euribor reference of this swap matures on the same date as the one of its fixed leg.

- To the contrary, a parallel shift of the par swap curve will have a minimal impact on the price of a Euribor/CMS swap, but a change in the slope of the swap curve can largely impact its valuation, since the maturity of the CMS reference extends beyond that of the CMS bond: As shown in the two charts following, in a steep yield environment, forward rates are priced at a higher yield than the spot curve;

- However, in a flat yield curve environment, forward rates are rather close to spot rates[45].

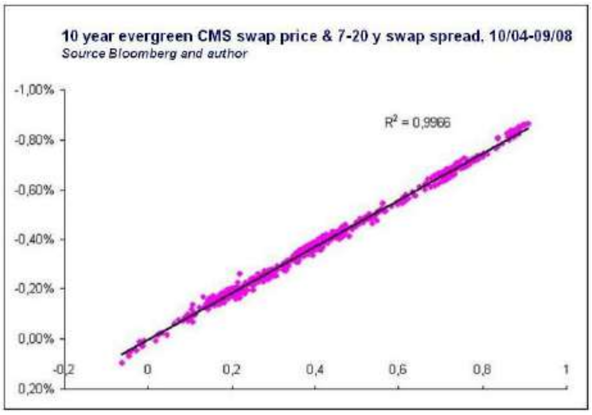
[45] The curve price sensitivity of CMS swaps is identical to the one of CMT or CMS bonds detailed in the 1st Volume of this document.





2.3.1 The shape of the swap curve constitutes the main component of CMS pricing

As explained in the 1st Volume of this document, the price of a CMS swap is strongly correlated to a curve spread representative of the shape of the curve. The chart opposite illustrates the strong correlation between the price of a 10 year "evergreen" CMS10 swap and the 7-20 year swap spread, using a CMS calculator from 10/2004 to 09/2008:



- We define as an "evergreen" swap a new CMS10 swap bearing always a 10 year maturity that we price every day against all the quarterly steps of the par curve,
- And compare its price with the 7-20 year swap spread

2.3.2 CMS swaps' quoted margin and discount margins

As already indicated paragraph 1.1.2, with a steep yield curve, the value of the CMS forward references is higher than that of its Euribor floating references and the yield of its Euribor references is also equal, on a same maturity, to the fixed rate yield of a Fixed rate/ Euribor par swap.

The quoted margin/ discount margin relationship in the floating rate world is equivalent to the fixed coupon/ yield relationship in the fixed rate world.

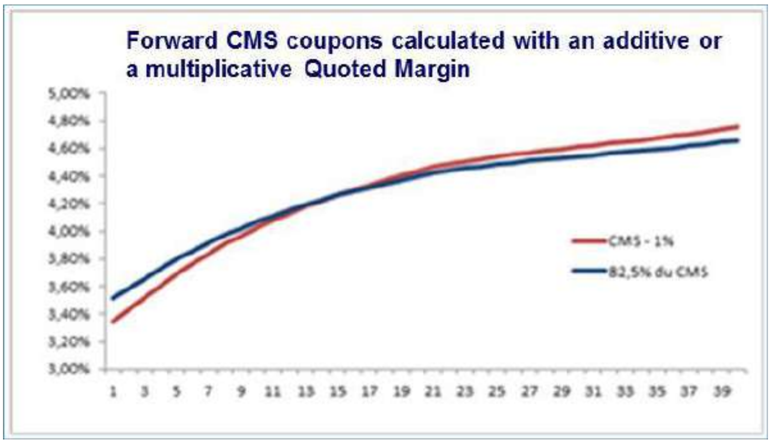
2.3.2.1 CMS swaps quoted margin

In order to equalize the yields of its Euribor floating leg and that of its CMS leg, with a steep yield curve, a fixed negative quoted margin is added to the CMS leg to produce a par CMS swap curve.

The CMS quoted margin can be applied either as an additive margin or a multiplicative margin:

- **Standard CMS swaps bear an additive quoted margin:** Like Euribor FRNs, CMS/ Euribor swaps pay usuallaly an additive margin that is directly added to their floating reference at each coupon fixing date. We described in the 1st volume of this document, Chapter 3, paragraph 2.4.2, why an additive margin offers to long term investors a better protection against a rise in long term interest rates.

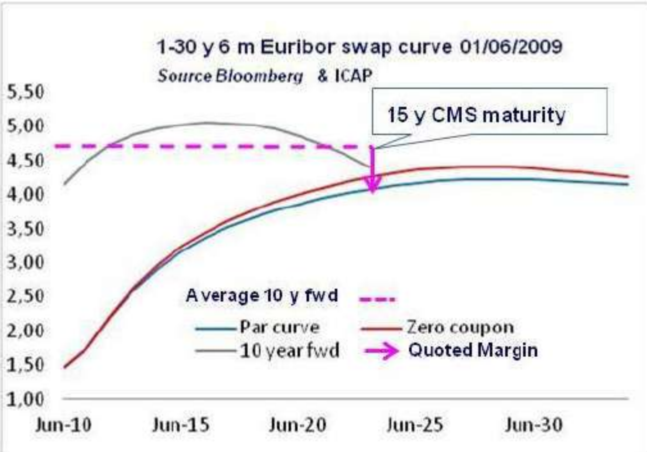
- **Taylor made CMS swaps can also bear a multiplicative quoted margin:** in such a case, the CMS pays a given percentage of its CMS reference. Many CMS structures sold to retail investors bear a multiplicative structure, since with a steep yield curve, these structures offer a better immediate current return compared to that of CMS structures paying an additive structure as shown on the chart below.



Quoted margin determination:

- The "indicative"[46] value of the quoted margin applicable to a not yet transacted par CMS/ Euribor swap, published by the interdealer brokers is constantly adjusted on their screens, in order to equalize the yield of its Euribor and CMS legs, whatever the changes in the shape of the par swap yield curve.

- The chart opposite illustrates how a fixed quoted margin is applied to the CMS10 leg of a 15 year CMS/ Euribor par swap, in order to equalize the value of its CMS leg yield with that of the fixed and floating legs of a Euribor par swap. In this example, the average value of the 10 year forward references is equal to 4.75% and the 15 year par swap is set at 4.17%. In a first approximation the quoted margin should be fixed at -58 bps:



- The Fixed value of the quoted margin is determined, "frozen" at the contractual conclusion of a new CMS/ Euribor swap and this fixed value quoted margin will apply to this specific CMS contract during its whole life.

[46] We qualify as "indicative" the Quoted Margin of a par swap that has not yet been transacted by opposition to the Fixed quoted Margin of an already transacted CMS swap.

The CMS model spread:

- In order to avoid any arbitrage opportunity, the value of the CMS leg should be equal to the value of its Euribor leg and consequently to the par swap rate applicable on the CMS swap maturity.

- In practice, when investors are keen to buy CMS bonds, a high level of demand can create a basis spread, like with currency swaps, between the theoretical price of the CMS and its market price. This spread is called the model spread of CMSs. However its value does not exceed in normal market conditions a few bps.

2.3.2.2 CMS swaps discount margin

After the fixation of the quoted margin, the CMS price evolves, depending on the changes in the slope of the par swap yield curve:

- In such a case, the fixed quoted margin does not express anymore the anticipated future yield of this secondary swap.
- To better analyze the yield of a CMS swap, the discount margin concept of an already transacted CMS swap compares the expected yield of this secondary swap with that of its reference, taking into account the last value of its floating reference, its fixed quoted margin, and the current market yield of a same Maturity par CMS swap.

2.4 CMS swaps' convexity price component

CMS fixed income instruments, like swaps or bonds indexed on a CMS or CMT reference, like the TEC 10, incorporate the price of an embedded option corresponding to the discrepancy between the periodicity of their coupons and the periodicity of their underlying reference. The valuation of this option has been largely described in the financial literature[47]. This embedded option is named the CMS convexity price adjustment:

- A 15 year CMS10/ Euribor swap pays every 3 month a coupon that corresponds to the fixing of its 10 year underlying reference determined at the beginning of each coupon period, i.e. 3 month earlier, from the inception of this CMS swap, up to its 15 year maturity.

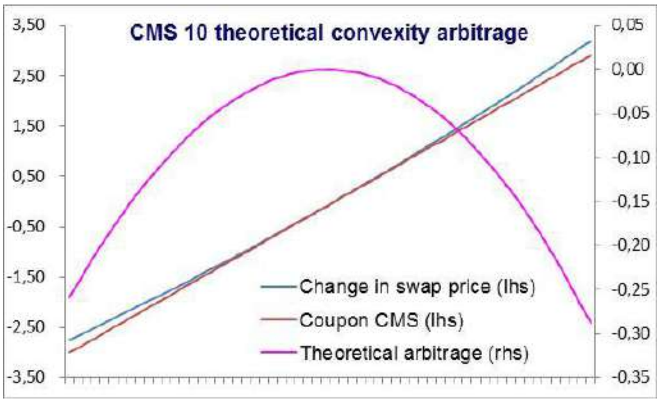
- However, the coupons of its 10 year underlying CMS reference are paid annually over a period of 10 years. Consequently, the convexity price of this 10 year reference that pays a 10 year unique coupon 3 months later on is negligible, whereas its 10 year reference swap bears a 10 year convexity price. For example, a CMS swap pays on 15 July 2007 a unique coupon of 4.5% corresponding to a 10 year swap reference that has been determined 3 month earlier in April 2007, where the 4.5% fixing published in April 2007 applies to a 10 year swap maturing on 15 April 2017, paying 10 yearly coupons of 4.5%.

[47] "Models for CMS caps" in "Euro Derivatives," Amblard, G. and Lebuchoux, J. Risk, September 2000. ftp://ftp.awl.co.uk/Longacre/SampleChaps/FTPH/0273654365.pdf
Swaption smile and CMS Adjustment, Fabio Mecurio, Banca IMI: http://www.fabiomercurio.it/RiskDRME.pdf

A trader hedging a CMS swap short position with no convexity[48] has to buy, according to an appropriate hedge ratio, a suite of 10 year par forward swaps that try to replicate the quarterly coupons of its CMS leg. But these 10 year forward swaps bear a 10 year convexity. Consequently, this strategy generates a synthetic portfolio that combines the initial CMS position and the forward swaps used as a hedge; this synthetic portfolio bears an embedded gamma negative position similar to being short options:

- We take the example of a CMS swap paying its coupons on 15 January, 15 April, 15 July and 15 October from 15 January 2004 to 15 January 2014; in order to hedge its CMS position, the trader had bought, on 10 May 2009, the 18 forward swaps that match its not yet fixed CMS coupons.

- On each subsequent fixing coupon dates, this trader, knowing the fixing of the next 10 year rate that he will pay 3 month later, reverses the hedge of this coupon by selling the specific 10 year forward swap that it had used to hedge this coupon. For instance the trader sells on 13 July 2011, the forward swap starting on 15 July 2011 and maturing 10 years later that hedged the CMS coupon that will be paid on 15 October 2011.



- In the case of a rate increase, the loss made by this trader on his sale of the swap is greater in absolute terms than the value of the CMS coupon that he pays,

- And in the case of a rate decrease, the value of the CMS coupon that it pays is greater, in absolute terms, than the selling price of the swap.

For those familiar with future contracts, this CMS price adjustment can be explained by analogy with the Euribor/ FRA convexity arbitrage that we have described in the 1st chapter of this document[49]; But hedging a CMS swap is by far more convex than hedging Euribor contracts.

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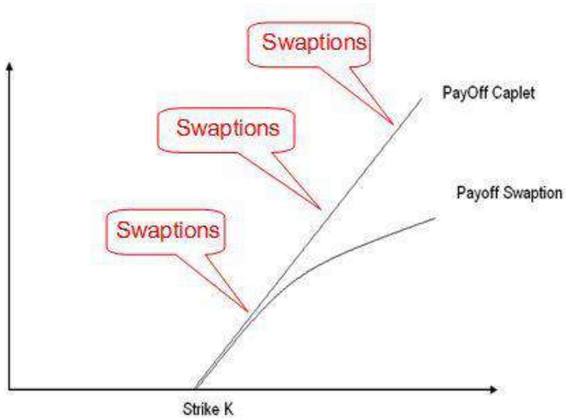
[48] The trader pays the CMS coupon.
[49] Euribor contracts have no convexity: Whatever their maturity, 1 tick is worth 12.5 euro. FRA contracts have some convexity, since they pay the net present value of 1 tick. In theory this should open arbitrage opportunities, since one may always gain on one instrument more than he loses of the other. So as to avoid this arbitrage, Euribor contracts incorporate the value of this embedded option.



The CMS convexity price component reflects the net price of the swaptions that this trader pays in the market to offset the value of the embedded option corresponding to the convexity price of the forward swaps used as a hedge.

- So as to adjust its hedge, the trader buys In The Money swaptions at various different strikes, for a nominal amount that will compensate the difference between the payoff of the CMS coupon and that of the forward par swaps used as a hedge and sells Out of The Money swaptions.

- The choice of these swaptions and their volatility smile adjustments are made by the investment banks swap desks active in the CMS market according to their own proprietary models that adjust the volatility smile of these in the money and out of the money swaptions; these models are regularly fitted to adapt to changes in markets' conditions.

2.5 The relative stability of the CMS swaps' convexity price component

The price of these swaptions expressed in percentage of at the money options depends on interest markets' volatility, but remains relatively stable under normal market circumstances and does not depends on the shape of the yield curve:

The chart following indicates

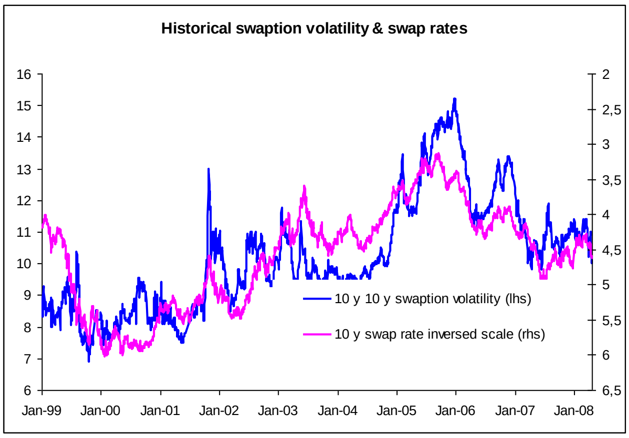
- On its left hand scale, the historical price of the convexity on CMS10 swaps bearing a 10, 12 and 15 year maturity. As shown on this chart, the convexity price component increases with the maturity of the CMS swap.

- And on its right hand scale the 7- 20 year swap spread that we use as an indicator of these CMS swaps, as already expressed paragraph 1.1.3.

For this reason we consider the value of the convexity component as an inelastic component of the CMS swap. Even with a rather flat curve, the value of the convexity price component remains stable. Most of the time, it represents a small proportion of the value of the discount margin, when expressed in bps. And in the case of a totally flat swap curve, the CMS the CMS discount margin would not be equal to zero due to the value of the convexity price component expressed in bps.

The relative stability of the convexity price component of the CMS swap under normal circumstances can be explained by the inverse correlation between 10 year in 10 year swaptions' volatility and the absolute interest rate levels, as shown on the opposite chart that compare the level of 10 year in 10 year swaptions' volatility, left hand scale, and the 10 year in 10 year swap rate, right hand scale.

On average from January 1999 to January 2009, the value of the convexity price component expressed in bps, the price in percentage of the CMS divided by its modified duration has represented between 15 and 20 bps on CMSs bearing a maturity between 10 and 15 years.



2.6 The influence of the CMS maturity on the level of the discount margin of a CMS10 swap

As already indicated, a CMS/ Euribor swap on one hand, and a Fixed rate/ Euribor swap on the other hand are priced against the same par swap curves published by interdealer brokers.

- Consequently, the Euribor legs of the CMS/ Euribor swap on one hand, and the Euribor leg of the Fixed rate/ Euribor swap have to offer the same fixed yield as the fixed leg of a Fixed rate/ Euribor swap.
- And the yield of a Fixed/ Euribor swap depends on its maturity, for instance to its 5, 10 or 15 years maturity; with a steep yield curve the yield to maturity of these Fixed rate/ Euribor swap increases.
- The table following compares the discount margin of 5, 10 & 15 year CMS reference to the CMS10. As indicated, with a steep yield curve the CMS discount margin decreases in absolute terms with the increased maturity of the CMS swap. For this reason, investors tend to invent in long CMS or CMT bonds, in order to reduce the value of the discount margin paid by these instruments.

Quoted Margin of 5, 10 & 15 year swap referenced on CMS10

	5 years	10 year	15 year
Swap rate	2,76%	3,46%	3,64%
Quoted Margin	-164,6	-124,2	-98,0

Source ICAP October 2009

3 Specific valuation of IRSs referenced to a sovereign constant maturity treasury rate

We illustrate the valuation of bonds paying a CMT reference, either a US CMT reference published by the FED or the French TEC 10 reference, on a theoretical TEC 10 2025 new OAT.

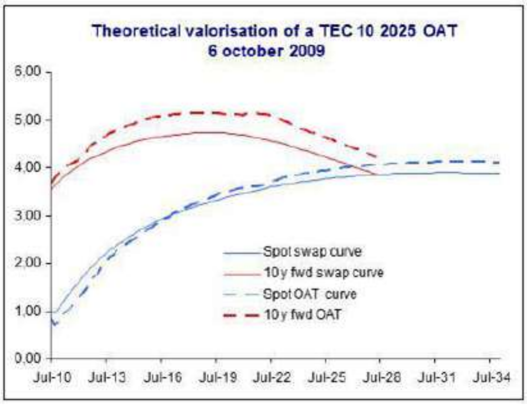
3.1 The current valuation of TEC10 swaps

Contrary to the still current belief of some investors, the valuation of a TEC 10 swap or bond paying a 3 month coupon is not any more based on the construction of the 10 year forward curve of fixed rate OATs. TEC 10 instruments are valorised against:

- The Fixed rate/ 6 month Euribor par curve,
- The 3/ 6 month Euribor basis swap,
- And a specific basis swap, the spread lock, applied to this calculation, as we will later on explain.

This change of market valuation is not new:

- At launch in April 1996 of the 1st TEC 10 2006 OAT, some traders having for many years arbitraged the OAT curve, were well aware of the specific reasons why some OATs where trading at cheaper or dearer market yields, when compared with the OAT theoretical yield curve.
- However other investment banks' traders continued, at that time, to price TEC10 OATs against the OAT yield curve; these traders sold unrealistic 10 year forward OAT asset swap spreads based on the yield of the then 10, 20 or 30 year spot yields, independently of the specific asset swap margin of these bonds. Some of these positions had to be closed later on by these traders at a high cost.
- Well before the current market turmoil, long term OAT yields incorporated, for several different reasons different asset swap spreads, for instance their above or below market prices and some specific liquidity and regulatory spreads.
- The current market turmoil has exacerbated the difference of valuation between the OAT curve and the swap curve; the OAT curve evidences the different liquidity cost of OATs bearing various long term maturities, where interest rate swaps prices, whatever their maturity, are based on the liquidity cost of an "evergreen" prime bank that rolls in the market with different counterparties a 6 month position over the life of the bond.
- As shown on the chart opposite that compares the 10 year forward curve calculated from the long term OAT yield and the long term par swap curve, using the OAT curves overestimates the yield of futures TEC10 coupons. Every 6 months AFT issue a new 10 year bond at an asset swap spread that is different from the asset swap spread of an old 30 year OAT bearing a residual maturity of 10 years.



- The CMS-TEC spread lock described below aims to solve this difference of pricing by contractually fixing the asset swap spread of forward 10 year fixed rate OATs.

3.2 The CMS-TEC spread lock

Since the market valorises TEC 10 instruments, against the fixed rate of a 6 month Euribor par swap curve, the pricing of TEC 10 instruments incorporates the additional anticipated basis swap between:

- The yield of the 10 year on the run OAT on each quarterly TEC 10 coupon fixing date,
- And the yield of a then 10 year par swap.

In the case of a 15 year TEC10/ CMS10 spread lock basis swap paying a 3 month coupon, the two counterparties exchange every 3 months the value of the 10 year TEC10 fixing and that of the 10 year swap fixing.

As shown on the ICAP Reuters page opposite, some interdealer brokers publish the indicative margin of this asset swap, but this market remains rather illiquid.

3.3 The CMS-TEC spread lock corresponds to a fixed rate component of a CMT swap

For the calculation of the price sensitivity of a CMS-TEC swap the value of this spread lock that constitutes a fixed rate component of this swap can be calculated, according to the change in market conditions, by reference to the Modified Duration formula.

4. Specific valuation of CMS or CMT bonds

4.1 The bond asset swap margin

Like any fixed income bonds launched by issuers bearing different signatures, CMS or CMT bonds are priced in the market according to their signature and pay consequently an asset swap margin that also constitute a fixed rate component of the bond, the being well suited to measure the price sensitivity of this CMS or CMT fixed rate component.

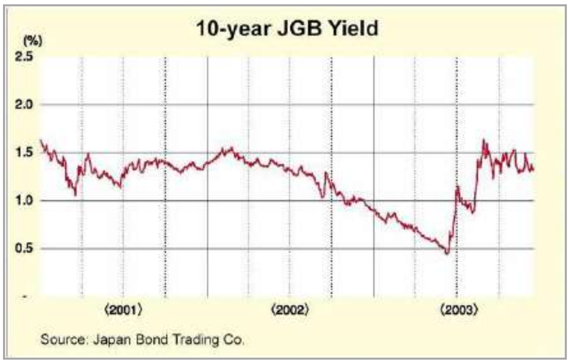
4.2 Value of the Cap or Floor options attached to a CMS or CMT bond

Plain vanilla CMS or CMT bonds can bear some optional components:

- Constant Maturity bonds paying an additive margin have to either explicitly or implicitly[50] incorporate a 0% floor, since as explained in the 1st Volume of this document, a bond cannot pay a negative coupon.

[50] Some CMT bonds, like TEC 10 OATs do not bear an explicit floor. However, lawyers usually estimate that a bond cannot pay a negative coupon contrary to interest swaps.

- Under normal market circumstances, the value of this floor is usually very low in a steep yield curve environment. However this floor will be in the money if the reference rate is fixed below the absolute value of the quoted margin, as it happened in the past with Japanese Floating Rate JGBs referenced to the 10 year JGB rate. Some of these issues pay an additive margin, superior in absolute value to 1% and the 10 year JGB fixed below 1% and as shown on the chart opposite in 2002 and 2003, 10 year fixed rate JGBs traded below 1%.



- Some other bonds can also incorporate a cap on the coupon paid by the floating rate bond, in order to increase its return. Swap traders valorising these caps and floors use the same forward volatility curve and smile adjustments as the one used to calculate the value of the convexity adjustment.

Chapter 4 – The price relationship between long term IRSs and other derivative instruments

As already explained in the introduction of this 2nd Volume, this very short chapter does not intend to describe in detail the characteristics and valuation methodology of all derivative instruments.

Its first paragraph briefly indicates the main characteristics of some largely used other derivatives and the second one recalls the very basic relationship existing between plain vanilla interest rate swaps and plain vanilla Caps, Floors and Swaptions.

1 Plain vanilla interest rate basis swaps and currency swaps

1.1 Plain vanilla basis swaps

1.1.1 Same currency basis swaps

These basis swaps pay two floating legs bearing a different index, for instance the 3 month/ 6 month Euribor basis swap or the CMS/ Euribor swap already described. The spread paid on these two legs can depend on different factors:

- The future anticipated level of their respective fixings due to the shape of the yield curve or a liquidity spread attached to their 3 or 6 month fixings, in the case of a Euribor basis swap

- And supply and demand factors like the Model Spread of CMS/ Euribor swaps described earlier.

Those two types of factors are often linked, a strong demand on long term Euribor references generating a higher fixing, as it is the case for instance with the 3 month/ 6 month Euribor basis swap.

1.1.2 Cross-currency basis swaps

Short term cross-currency swaps defined as a spot sale and a forward purchase of a pair of currencies between the two same counterparties, that are at the origin of the derivative markets, are largely used in the interbank market to exchange liquidities in different currencies.

Long term cross-currency swaps are also very commonly used in the market. For instance a corporate issuer having launched a 300 million 5 years EUR fixed rate bond can transform its exposure into a synthetic liability in USD, by entering into two different interest rate swaps:

- A plain vanilla EUR interest rate swap where the issuer receives the 5 year fixed rate and pays 3 month Euribor plus a contractual margin,

- And a 3 month EUR/ 3 month USD Libor cross-currency swap where the issuer pays 3 month Libor plus a margin and receives 3 month Euribor. Contrary to same currency interest rate swaps, cross-currency interest rate swaps exchange, at the spot exchange rate prevailing in the market, their respective nominal amounts at the swap intuition and this exchange rate prevailing at the intuition is also used for the exchange of nominal amounts at the cross-currency swap maturity.

For many years, the margin applied to cross currency swap was only dictated by a supply and demand effect.

However as shown on the chart opposite, since August 2007, the valuation of cross-currency swaps have largely changed, due to the high demand of USD by European banks.

- In December 2004, a 5 year EUR/ USD basis could be exchanged for a spread of around 3 bps, where in October 2011, a bank willing to convert its Euribor borrowing into USD Libor had to pay a spread of 40 bps on a 5 year maturity. A bank having issued at Euribor plus 50 bps in € will after the conclusion of this cross-currency swap pay its USD Libor funding at a total cost of 90 bps.

2. Swaptions, caps, floors and collar and swap price relationship

This brief paragraph recalls the very basic relationship existing between plain vanilla interest rate swaps and plain vanilla Caps, Floors and Swaptions, since some market participants seem to ignore those basic concepts.

2.1. Swaptions

A swaption, like the options traded by a swap desk to hedge the convexity of the CMS leg of a CMS/ Euribor swap described earlier, is a contractual contract between two counterparties where the buyer receives the right to pay, for a given rate and notional amount and at a given date, the fixed rate of a forward swap transaction. This option is defined as a call option; a put option gives to its buyer the right to receive the forward rate of a swaption.

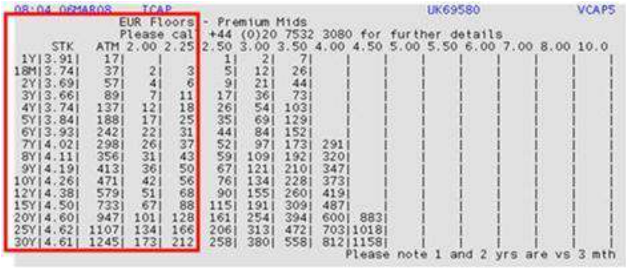
A swaption is at the money, if its strike rate is equal to the forward rate of its underlying forward swap and in the option Put-Call parity theory, the value of an at the money call option is equal to the value of an at the money put option.

2.2. Caps, floor and collar

2.2.1 The Put-Call parity of swaptions applies also to the value of at the money caps and floors

A cap limits to a contractual maximum strike rate the level of interest rate that a borrower will pay on its floating rate borrowing, for instance 3 month Euribor, for a given amount and a given period, for instance 5 years. If at any fixing date of this borrowing the fixing is higher than the contractual strike rate, the seller of the cap pays to the buyer, the borrower, the difference of interest between this fixing rate and the strike rate. Similarly a floor limits to a minimum rate the level of interest that a borrower will pay on its borrowing.

The following ICAP Reuter pages illustrate the equal par price of swaption premium paid according to their maturity on at the money caps and floors.



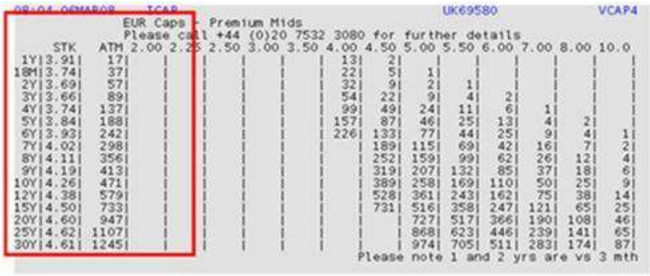
08:04 06MAR08 ICAP UK69580 VCAP5

EUR Floors - Premium Mids

Please call +44 (0)20 7532 3080 for further details

	STK	ATM	2.00	2.25	2.50	3.00	3.50	4.00	4.50	5.00	5.50	6.00	7.00	8.00	10.0
1Y	3.91	17			1	2	7								
18M	3.74	37	2	3	5	12	26								
2Y	3.69	57	4	6	9	21	44								
3Y	3.66	89	7	11	17	36	73								
4Y	3.74	137	12	18	26	54	103								
5Y	3.84	188	17	25	35	69	129								
6Y	3.93	242	22	31	44	84	152								
7Y	4.02	298	26	37	52	97	173	291							
8Y	4.11	356	31	43	59	109	192	320							
9Y	4.19	413	36	50	67	121	210	347							
10Y	4.26	471	42	56	76	134	228	373							
12Y	4.38	579	51	68	90	155	260	419							
15Y	4.50	733	67	88	115	191	309	487							
20Y	4.60	947	101	128	161	254	394	600	883						
25Y	4.62	1107	134	166	206	313	472	703	1018						
30Y	4.61	1245	173	212	258	380	558	812	1158						

Please note 1 and 2 yrs are vs 3 mth



08:04 06MAR08 ICAP UK69580 VCAP4

EUR Caps - Premium Mids

Please call +44 (0)20 7532 3080 for further details

	STK	ATM	2.00	2.25	2.50	3.00	3.50	4.00	4.50	5.00	5.50	6.00	7.00	8.00	10.0
1Y	3.91	17						13	2						
18M	3.74	37						22	5	1					
2Y	3.69	57						32	9	2	1				
3Y	3.66	89						54	22	9	4	2			
4Y	3.74	137						99	49	24	11	6	1		
5Y	3.84	188						157	87	46	25	13	4	2	
6Y	3.93	242						226	133	77	44	25	9	4	1
7Y	4.02	298							189	115	69	42	16	7	2
8Y	4.11	356							252	159	99	62	26	12	4
9Y	4.19	413							319	207	132	85	37	18	6
10Y	4.26	471							389	258	169	110	50	25	9
12Y	4.38	579							528	361	243	162	75	38	14
15Y	4.50	733							731	516	358	247	121	65	25
20Y	4.60	947								727	517	366	190	108	46
25Y	4.62	1107								868	623	446	239	141	65
30Y	4.61	1245								974	705	511	283	174	87

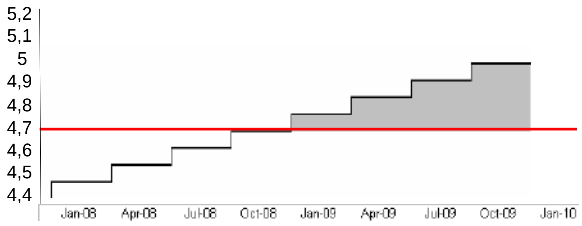
Please note 1 and 2 yrs are vs 3 mth

2.2.2 But the at the money price of caps and floors is often misunderstood

A cap or a floor bear an unique strike price, where the forward value of each of their constituting caplets, the underlying forward rate of each future fixing, bear a different price and many market participants estimate that the at the money strike price of caps and floors is well defined.

[Under IFRS accounting for instance a cap was considered as out of the money if the cap strike price was superior to the market rate of its 1st caplet.]

The Euribor floating leg yield of a Fixed rate/ Euribor swap is equal to the yield of its fixed leg and consequently, the at the money strike rate of caps and floors is equal to the fixed rate of its underlying swap rate.



The two 1st charts opposite illustrate different collar strategy:

- The maximum and minimum strike rate of a collar can be rather large or much more narrow, depending on the borrower strategy
- And if, as in the 3rd chart the strike rate of the cap is equal to the strike rate of the floor, then this collar is simply an interest rate swap.